

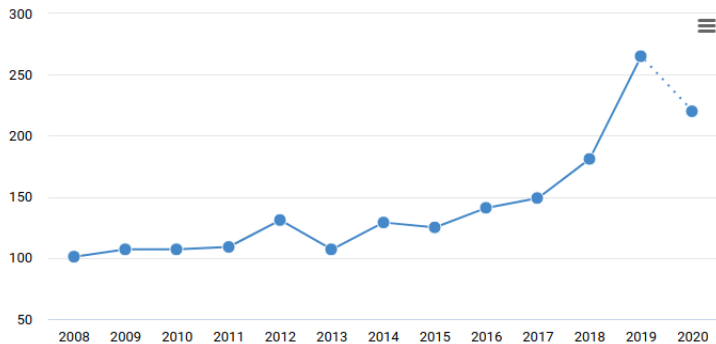
The Zero-Knowledge Proof Revolution

interesting insights about

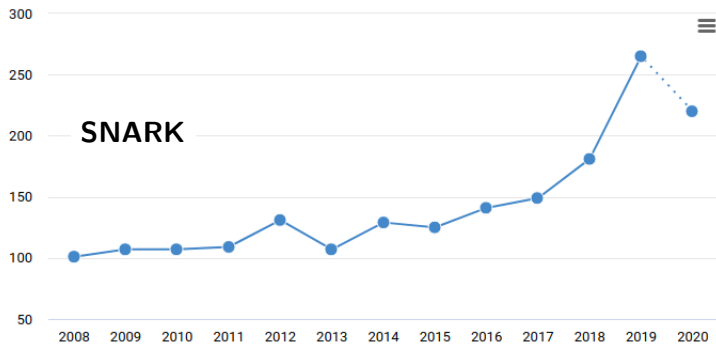
Alan Szepieniec
alan@asdm.gmbh



Revolution

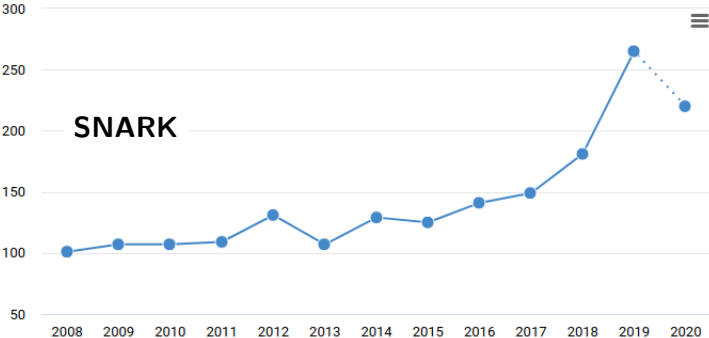


Revolution



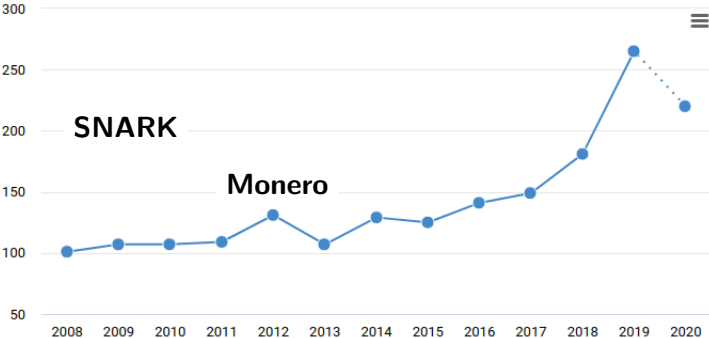
Revolution

ZCash

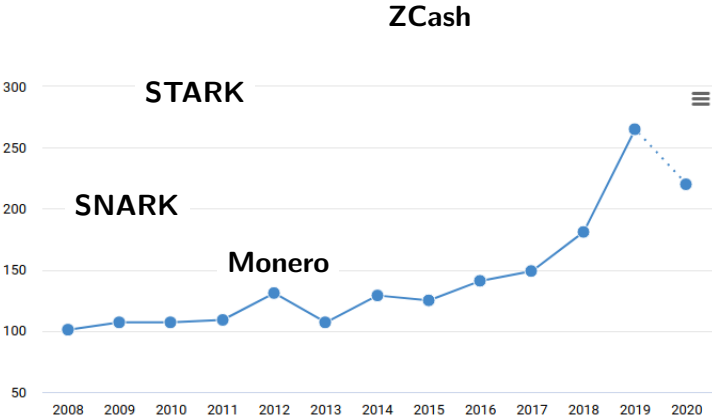


Revolution

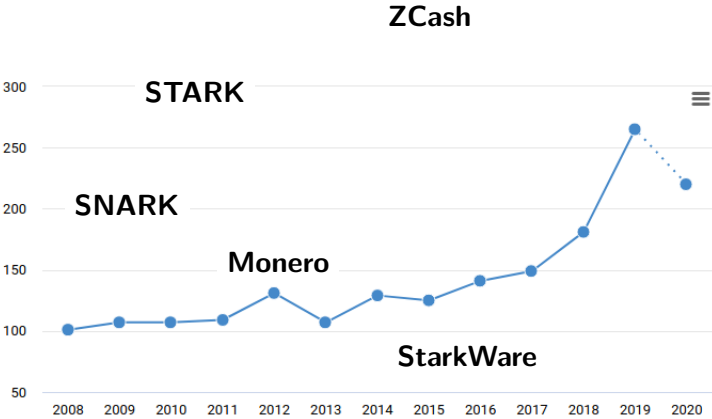
ZCash



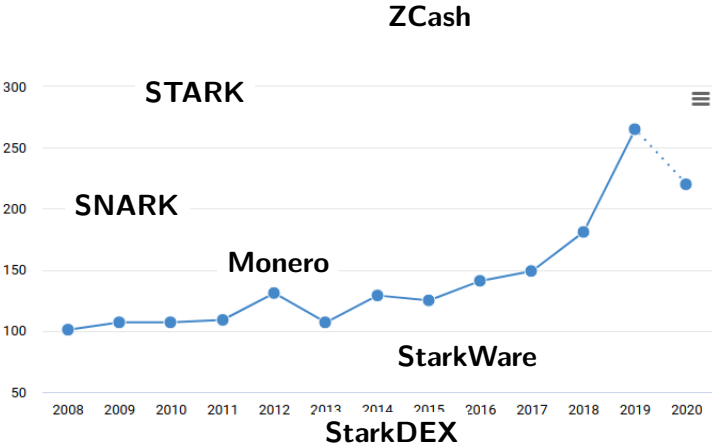
Revolution



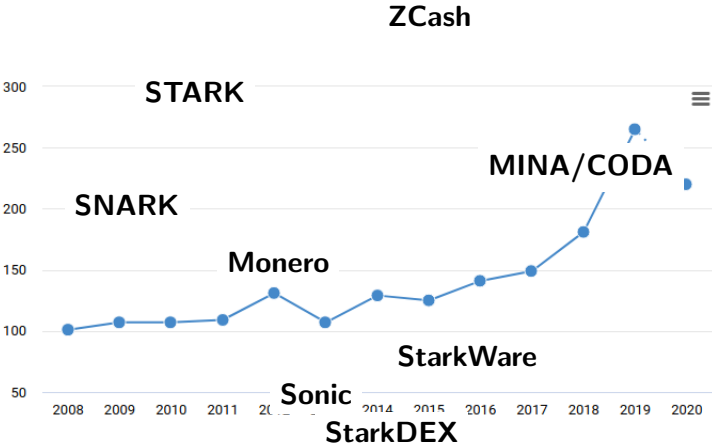
Revolution



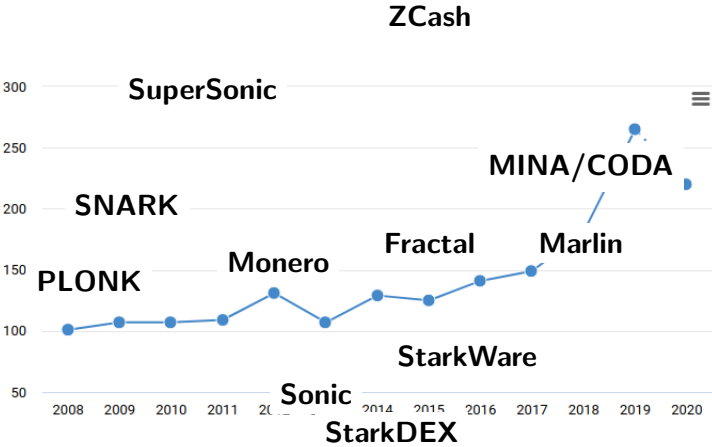
Revolution



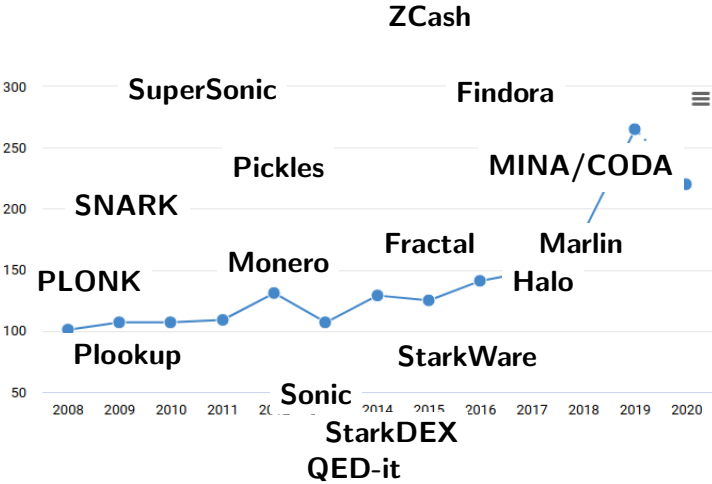
Revolution



Revolution



Revolution



Questions

Why is there a ZKP Revolution?

What does it mean for
me / Bitcoin / cryptocurrencies?

~~How do ZKPs work?~~

What can ZKPs achieve?

What does the future hold?

Questions

Why is there a ZKP Revolution?

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1. Philosophy

~~How do ZKPs work?~~

What can ZKPs achieve?

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Questions

Why is there a ZKP Revolution?

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1. Philosophy

2. Technology

~~How do ZKPs work?~~

What can ZKPs achieve?

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Questions

Why is there a ZKP Revolution?

1. Philosophy

What does it mean for
me / Bitcoin / cryptocurrencies?

2. Technology

~~How do ZKPs work?~~

What can ZKPs achieve?

3. Applications

What does the future hold?



1. PHILOSOPHY

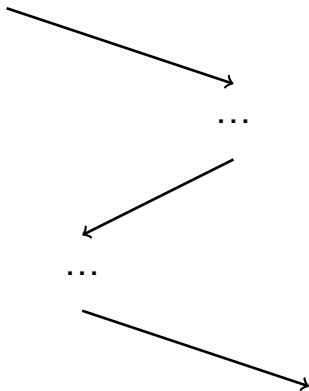
Motivation

Motivation

what is money?

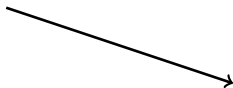
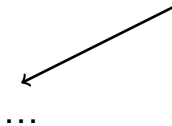
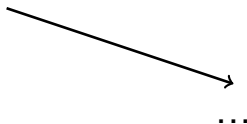
Motivation

what is money?



Motivation

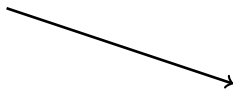
what is money?



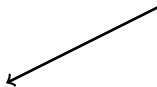
cryptography

Motivation

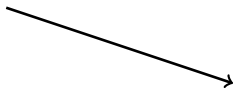
what is money?



...



...

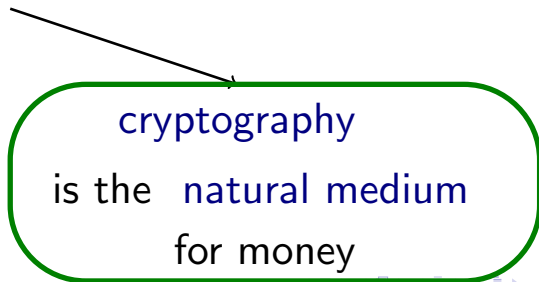
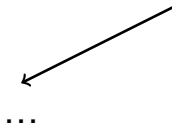
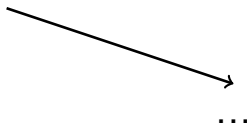


cryptocurrency

is the natural medium
for money

Motivation

what is money?



What is Money?

What is Money?

what you buy stuff with

What is Money?

✓ practical

what you buy stuff with

x recursive

What is Money?

✓ practical

what you buy stuff with

x recursive

{
medium of exchange
store of value
unit of account
}

What is Money?

✓ practical

what you buy stuff with

x recursive

⎧ medium of exchange store of value unit of account ⎫	✓ functions
	x not properties

What is Money?

✓ practical

what you buy stuff with
x recursive

{	medium of exchange	}	✓ functions
	store of <u>value</u>		x not properties
	unit of account		x subjective

What is Money?

✓ practical

what you buy stuff with
x recursive

{	medium of exchange	}	✓ functions
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the most marketable good

What is Money?

✓ practical

what you buy stuff with
x recursive

{	medium of exchange	}	✓ functions
	store of <u>value</u>		x not properties
	unit of account		x subjective

the most marketable [✓] good [✓]

What is Money?

✓ practical

what you buy stuff with
x recursive

⎧ medium of exchange store of <u>value</u> unit of account ⎫	✓ functions
	x not properties
	x subjective

the most marketable good ✓
~ context-dependent ✓

What is Money?

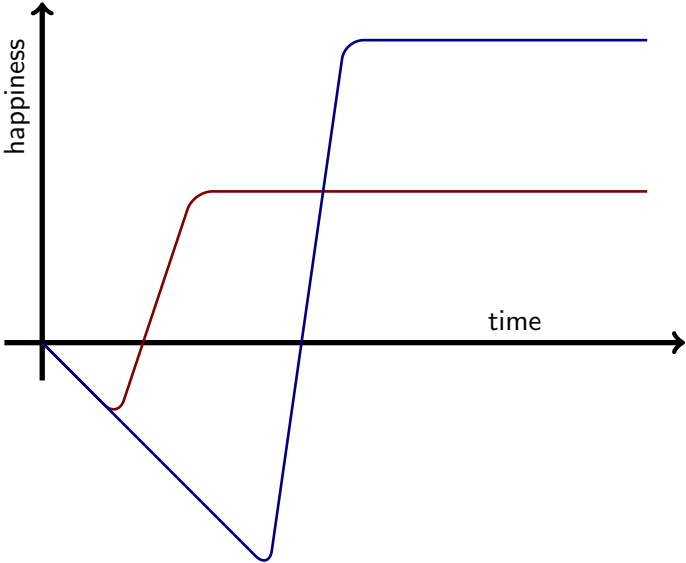
✓ practical

what you buy stuff with
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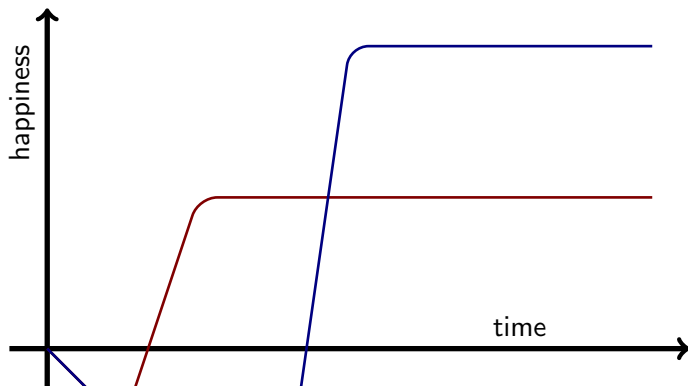
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the most marketable [✓]good [✓]
~ context-dependent
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Robinson Crusoe

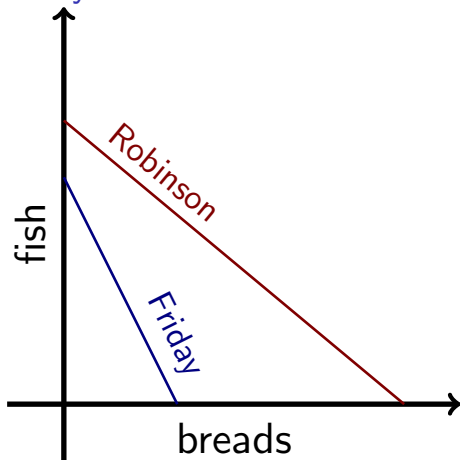


Robinson Crusoe

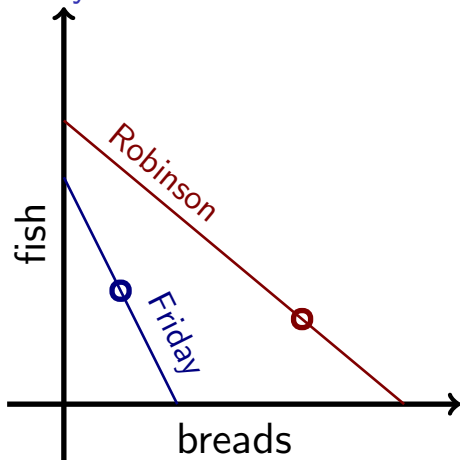


✓ economize
✓ capital investment
x money

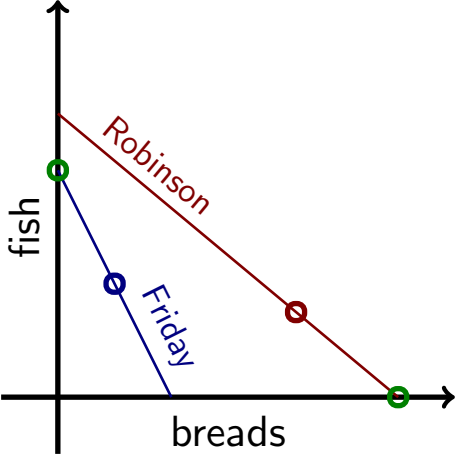
Robinson and Friday



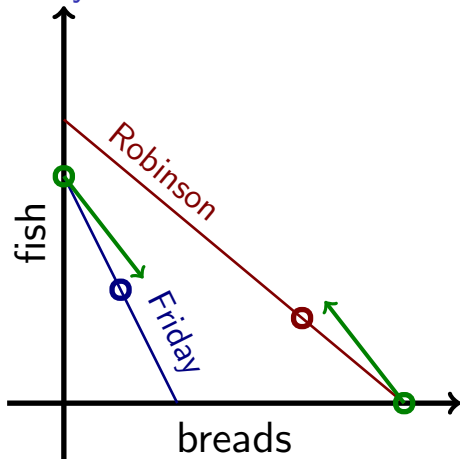
Robinson and Friday



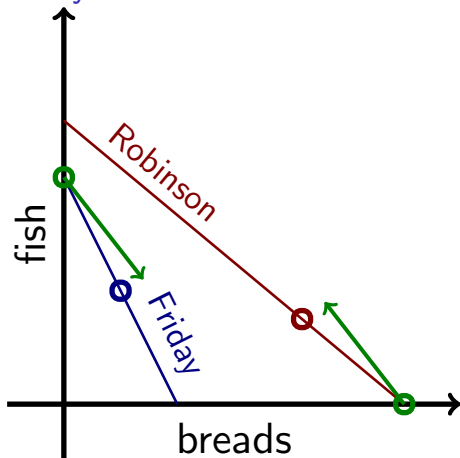
Robinson and Friday



Robinson and Friday



Robinson and Friday



✓ division of labor

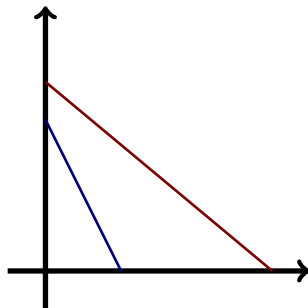
✓ ~ trust

x money

Larger Economies Need Money

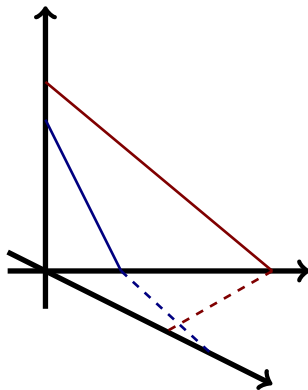
Larger Economies Need Money

trust



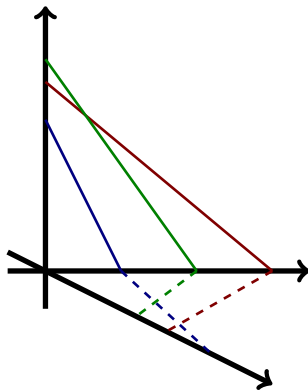
Larger Economies Need Money

trust
coincidence of wants



Larger Economies Need Money

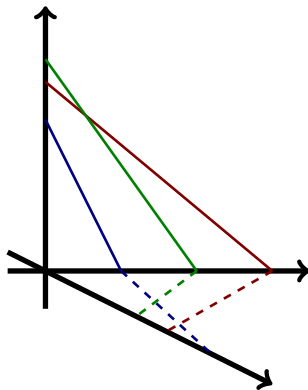
trust
coincidence of wants
coordination



Larger Economies Need Money

trust
coincidence of wants
coordination

↑
money fixes this



Larger Economies Need Money

trust

coincidence of wants

coordination

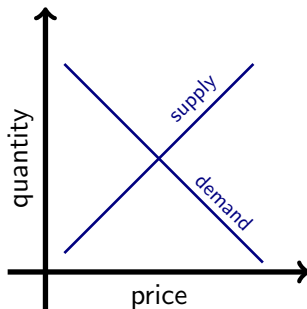


money fixes this

Larger Economies Need Money

trust
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↑
money fixes this



Larger Economies Need Money

trust

coincidence of wants

coordination



money fixes this

Larger Economies Need Money

trust —————→ adversarial context ✓

coincidence of wants

coordination



money fixes this

Larger Economies Need Money

trust —————→ adversarial context ✓

coincidence of wants —————→ medium of exchange ✓

coordination



money fixes this

Larger Economies Need Money

trust —————→ adversarial context ✓

coincidence of wants —————→ medium of exchange ✓

coordination —————→ carrier of information ✓

↑
money fixes this

Larger Economies Need Money

trust —————→ adversarial context ✓

coincidence of wants —————→ medium of exchange ✓

coordination ———→ carrier of information ✓

↑
money fixes this

↑
cryptography fixes this

Objectives of Cryptography

Objectives of Cryptography

CONFIDENTIALITY

INTEGRITY

AUTHENTICITY

Objectives of Cryptography

CONFIDENTIALITY

INTEGRITY

AUTHENTICITY

for money?

Objectives of Cryptography

~~CONFIDENTIALITY~~

INTEGRITY

AUTHENTICITY

for money?

Objectives of Cryptography

~~CONFIDENTIALITY~~

INTEGRITY

~~AUTHENTICITY~~

for money?

Objectives of Cryptography

~~CONFIDENTIALITY~~

INTEGRITY

VERIFIABILITY ✓ ✓ ✓

~~AUTHENTICITY~~

for money?

Objectives of Cryptography

~~CONFIDENTIALITY~~

INTEGRITY ← VERIFIABILITY ✓ ✓ ✓

~~AUTHENTICITY~~

for money?

2. BASICS of ZERO-KNOWLEDGE PROOFS

Proof Systems in Mathematics

Proof Systems in Mathematics

in theory:

mathematician

mathematician

Proof Systems in Mathematics

in theory:

proposition



mathematician

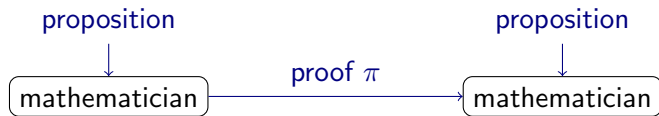
proposition



mathematician

Proof Systems in Mathematics

in theory:



Proof Systems in Mathematics

in theory:



Proof Systems in Mathematics

in theory:



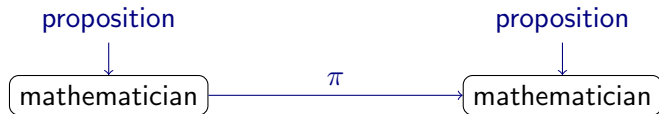
in practice:

Proof Systems in Mathematics

in theory:



in practice:

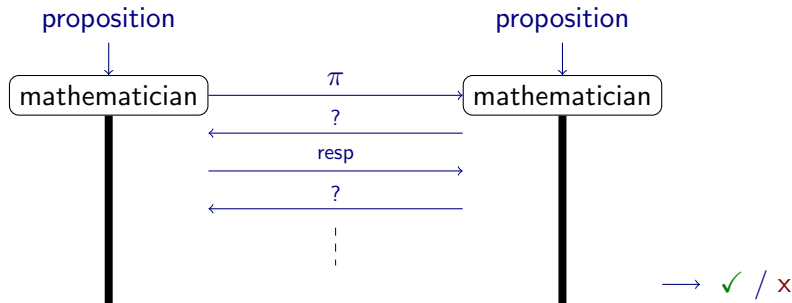


Proof Systems in Mathematics

in theory:



in practice:

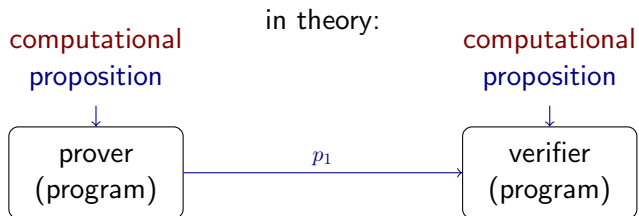


Proof Systems in Computer Science

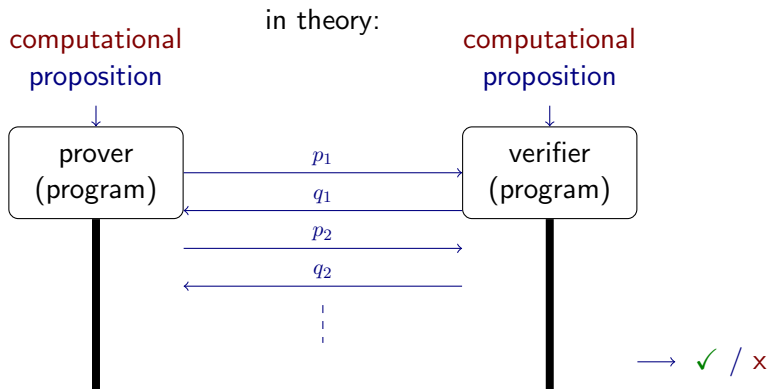
Proof Systems in Computer Science

in theory:

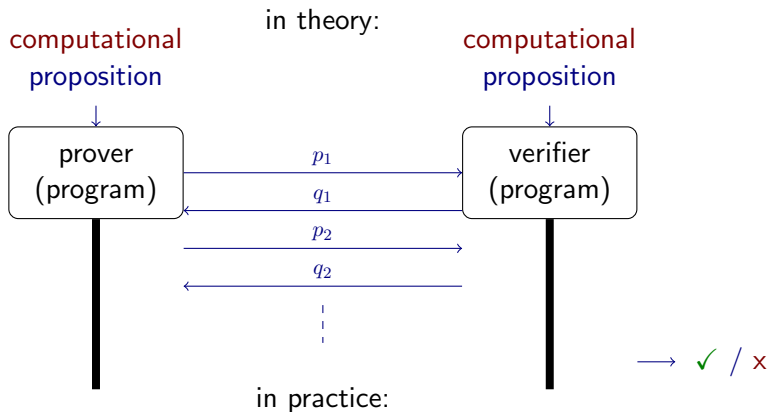
Proof Systems in Computer Science



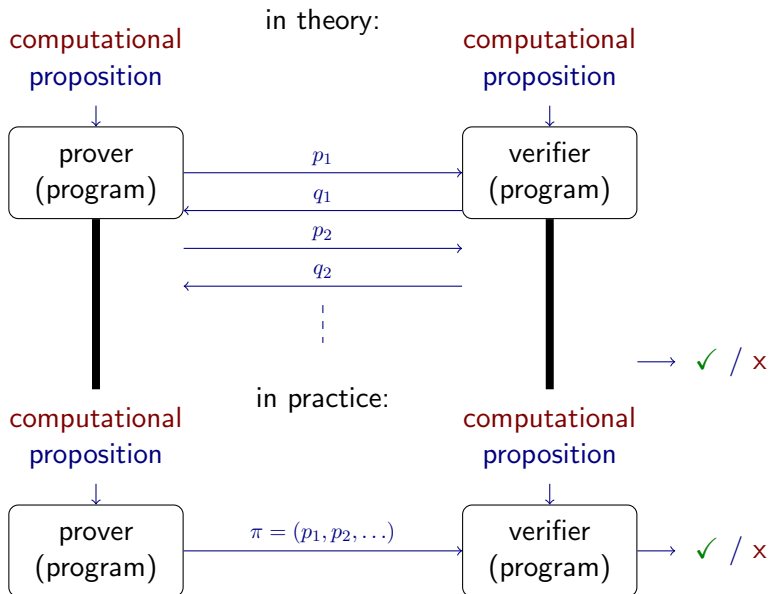
Proof Systems in Computer Science



Proof Systems in Computer Science



Proof Systems in Computer Science



Computational Propositions

Computational Propositions

Program *A*

Computational Propositions

Program A

- takes public input x

Computational Propositions

Program A

- takes public input x
- takes secret input y

Computational Propositions

Program A

- takes **public input x**
- takes **secret input y**
- outputs **public value z**

Computational Propositions

Program A

- takes public input x
- takes secret input y
- outputs public value z
- runs in time T

Computational Propositions

Program A

- takes **public input x**
- takes **secret input y**
- outputs **public value z**
- runs in **time T**

Naïve Verifier (x, y, z):

- run $A(x, y)$ for T steps
- test output against z

Computational Propositions

Program A

- takes public input \mathbf{x}
- takes secret input \mathbf{y}
- outputs public value \mathbf{z}
- runs in time T

Naïve Verifier $(\mathbf{x}, \mathbf{y}, \mathbf{z})$:

- run $A(\mathbf{x}, \mathbf{y})$ for T steps
- test output against \mathbf{z}

Prover (\mathbf{x}, \mathbf{y})

- generate π
- while running $\mathbf{z} \leftarrow A(\mathbf{x}, \mathbf{y})$

Computational Propositions

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Resource-Constrained Verifier $(\mathbf{x}, \mathbf{z}, \pi)$

- no access to secrets
- running time $t \ll T$

Computational Propositions

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- generate π
- while running $\mathbf{z} \leftarrow A(\mathbf{x}, \mathbf{y})$

Resource-Constrained Verifier $(\mathbf{x}, \mathbf{z}, \pi)$

- no access to secrets \longrightarrow zero-knowledge proof
- running time $t \ll T$ \longrightarrow succinct proof (SNARK)

Succinct and Zero-Knowledge — Intuition

Succinct and Zero-Knowledge — Intuition

				2				
							1	
5								
	8							
					4			
								7
						3		
			6					
		9						

Succinct and Zero-Knowledge — Intuition

				2				
							1	
5								
	8							
					4			
								7
						3		
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		9						

Succinct and Zero-Knowledge — Intuition

				2				
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5								
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					4			
								7
						3		
			6					
		9						

1. select row, column, or block

Succinct and Zero-Knowledge — Intuition

				2				
							1	
5								
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					4			
								7
						3		
			6					
		9						

1. select row, column, or block
2. shuffle

Succinct and Zero-Knowledge — Intuition

				2				
							1	
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		9						

1. select row, column, or block
2. shuffle
3. flip and check

Succinct and Zero-Knowledge — Intuition

				2				
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						3		
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		9						



1. select row, column, or block
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3. flip and check

III Applications

III Applications A) to cryptocurrencies

Schnorr

Schnorr zero-knowledge proof

secret key: $x \in \mathbb{Z}_p$

public key: $\mathbf{P} = x \times \mathbf{G}$

Schnorr

Schnorr zero-knowledge proof

secret key: $x \in \mathbb{Z}_p$

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protocol:

\mathcal{P}

\mathcal{V}

Schnorr

Schnorr zero-knowledge proof

secret key: $x \in \mathbb{Z}_p$

public key: $\mathbf{P} = x \times \mathbf{G}$

protocol:

\mathcal{P} \mathcal{V}

$r \xleftarrow{\$} \mathbb{Z}_p$

$\mathbf{R} \leftarrow r \times \mathbf{G}$

\mathbf{R}
—————→

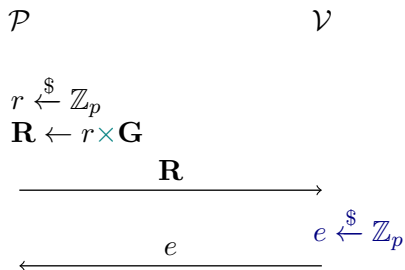
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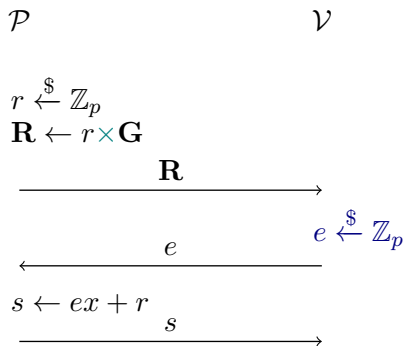
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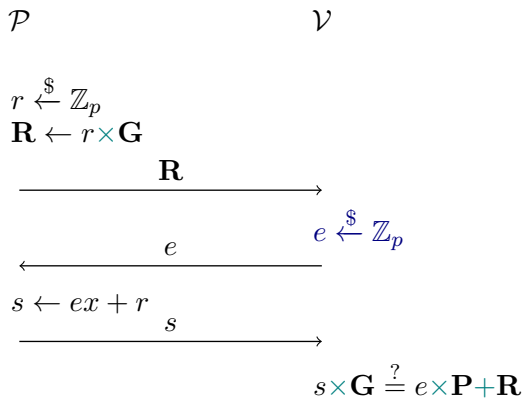
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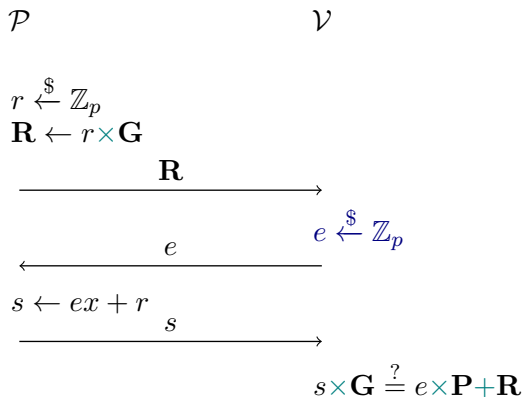
Schnorr

Schnorr zero-knowledge proof signature scheme

secret key: $x \in \mathbb{Z}_p$

public key: $\mathbf{P} = x \times \mathbf{G}$

protocol:



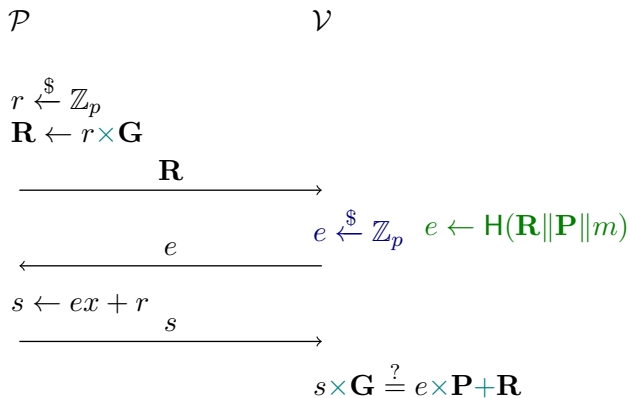
Schnorr

Schnorr zero-knowledge proof signature scheme

secret key: $x \in \mathbb{Z}_p$

public key: $\mathbf{P} = x \times \mathbf{G}$

protocol:



Schnorr

Schnorr zero-knowledge proof

signature scheme

signature: (\mathbf{R}, s)

secret key: $x \in \mathbb{Z}_p$

public key: $\mathbf{P} = x \times \mathbf{G}$

protocol:

\mathcal{P}

\mathcal{V}

$$r \xleftarrow{\$} \mathbb{Z}_p$$

$$\mathbf{R} \leftarrow r \times \mathbf{G}$$

$$\xrightarrow{\mathbf{R}}$$

$$e \xleftarrow{\$} \mathbb{Z}_p \quad e \leftarrow \mathbf{H}(\mathbf{R} \parallel \mathbf{P} \parallel m)$$

$$\xleftarrow{e}$$

$$s \leftarrow ex + r$$

$$\xrightarrow{s}$$

$$s \times \mathbf{G} \stackrel{?}{=} e \times \mathbf{P} + \mathbf{R}$$

Multisig Protocol

Multisig Protocol

	sk	pk	sig
Alice	x_a	$x_a \times \mathbf{G}$	$(r_a \times \mathbf{G}, ex_a + r_a)$

Multisig Protocol

	sk	pk	sig
Alice	x_a	$x_a \times \mathbf{G}$	$(r_a \times \mathbf{G}, ex_a + r_a)$
Bob	x_b	$x_b \times \mathbf{G}$	$(r_b \times \mathbf{G}, ex_b + r_b)$

Multisig Protocol

	sk	pk	sig
Alice	x_a	$x_a \times \mathbf{G}$	$(r_a \times \mathbf{G}, e x_a + r_a)$
Bob	x_b	$x_b \times \mathbf{G}$	$(r_b \times \mathbf{G}, e x_b + r_b)$
joint	$x_a + x_b$	$(x_a + x_b) \times \mathbf{G}$	$((r_a + r_b) \times \mathbf{G}, e(x_a + x_b) + r_a + r_b)$

Multisig Protocol

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joint	$x_a + x_b$	$(x_a + x_b) \times \mathbf{G}$	$((r_a + r_b) \times \mathbf{G}, e(x_a + x_b) + r_a + r_b)$

$$e = H((\mathbf{R}_a + \mathbf{R}_b) \parallel \mathbf{P} \parallel m)$$

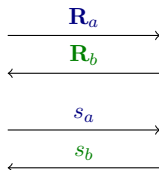
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Alice	x_a	$x_a \times \mathbf{G}$	$(r_a \times \mathbf{G}, ex_a + r_a)$
Bob	x_b	$x_b \times \mathbf{G}$	$(r_b \times \mathbf{G}, ex_b + r_b)$
joint	$x_a + x_b$	$(x_a + x_b) \times \mathbf{G}$	$((r_a + r_b) \times \mathbf{G}, e(x_a + x_b) + r_a + r_b)$

Alice

Bob

$$e = H((\mathbf{R}_a + \mathbf{R}_b) || \mathbf{P} || m)$$



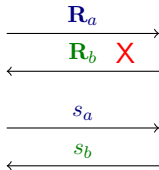
Multisig Protocol

	sk	pk	sig
Alice	x_a	$x_a \times \mathbf{G}$	$(r_a \times \mathbf{G}, ex_a + r_a)$
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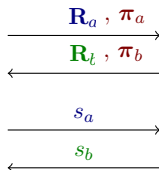
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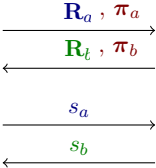
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Alice

Bob

$$e = H((\mathbf{R}_a + \mathbf{R}_b) || \mathbf{P} || m)$$



honest-but-curious security $\xrightarrow{\text{ZKPs}}$ malicious security

Zero-Knowledge Contingent Payments

Zero-Knowledge Contingent Payments

Alice
(information)

Bob
(\$\$\$)

Zero-Knowledge Contingent Payments

Alice
(information)



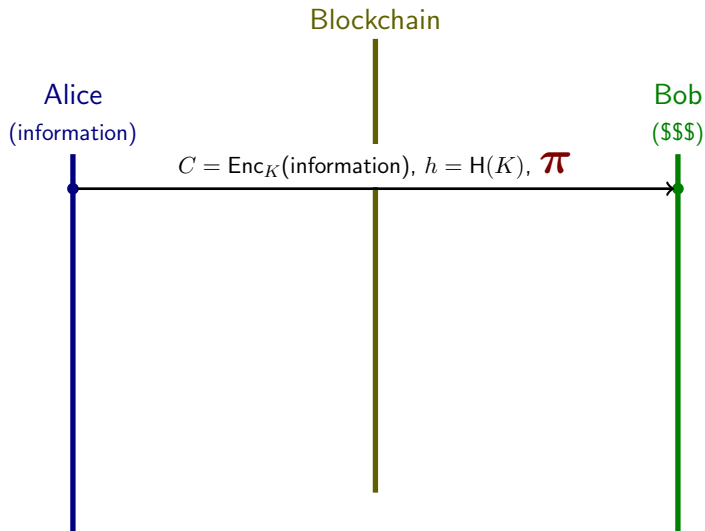
Blockchain



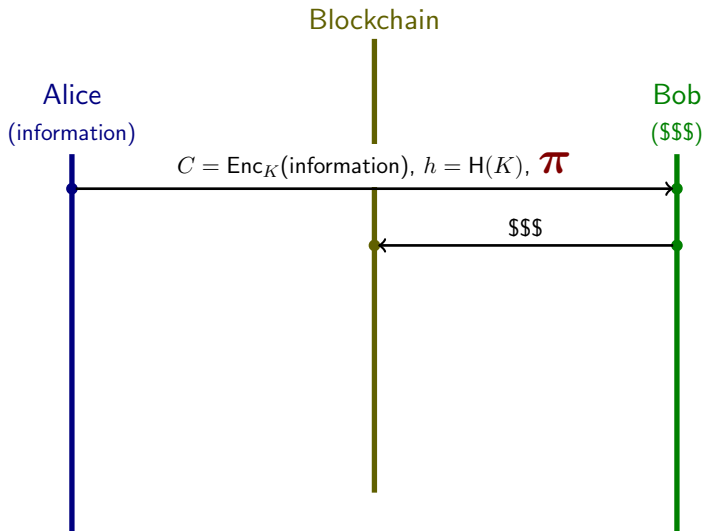
Bob
(\$\$\$)



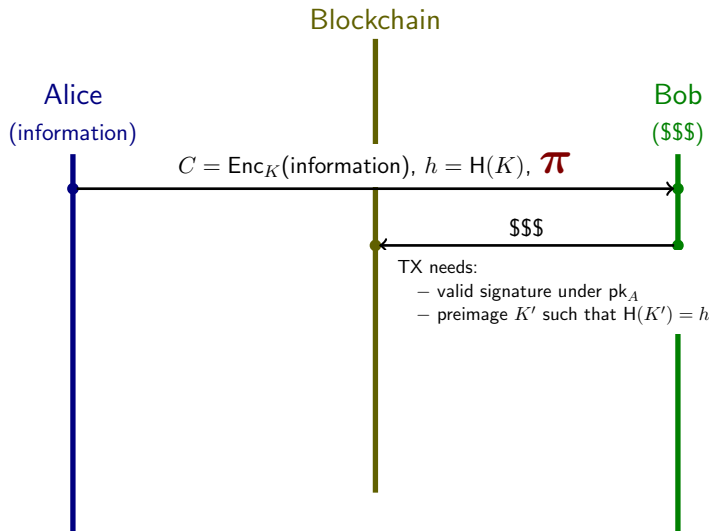
Zero-Knowledge Contingent Payments



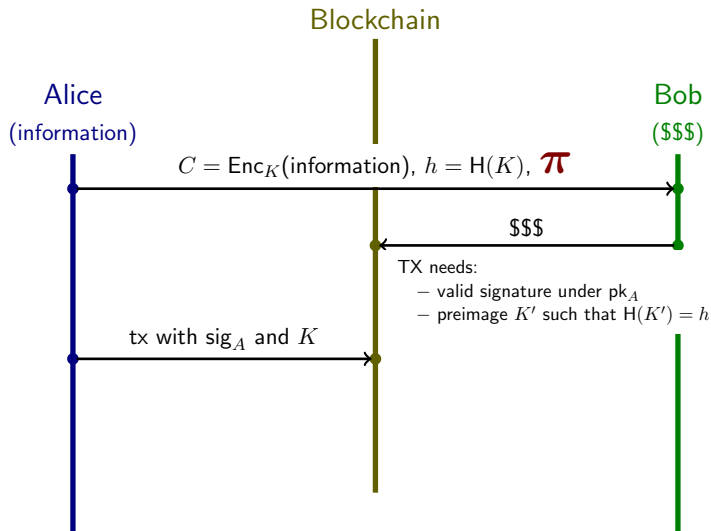
Zero-Knowledge Contingent Payments



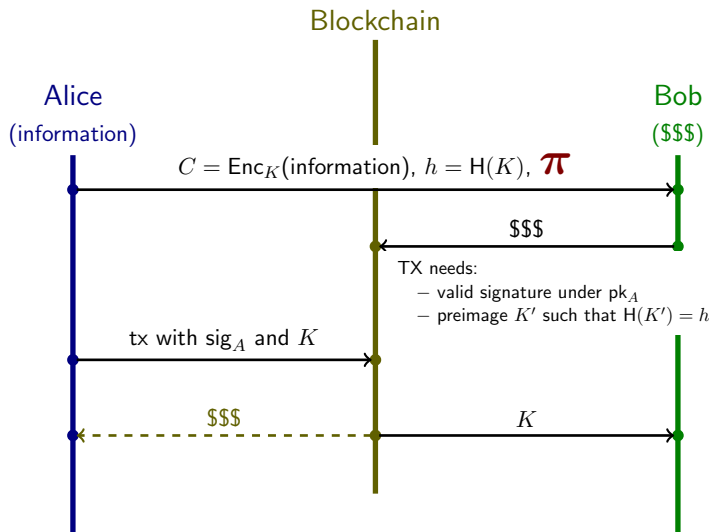
Zero-Knowledge Contingent Payments



Zero-Knowledge Contingent Payments

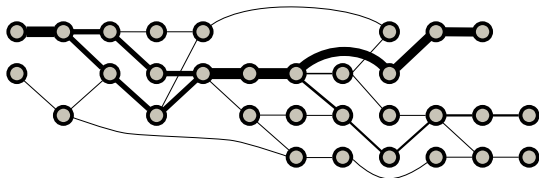


Zero-Knowledge Contingent Payments



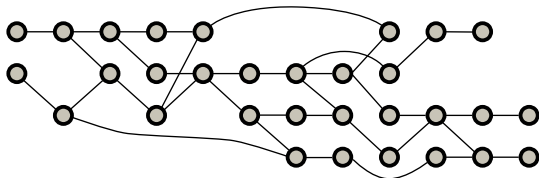
Privacy Coins

Privacy Coins



no privacy

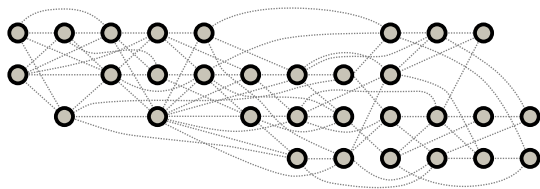
Privacy Coins



confidential transactions

π \leftrightarrow "amounts are positive"

Privacy Coins



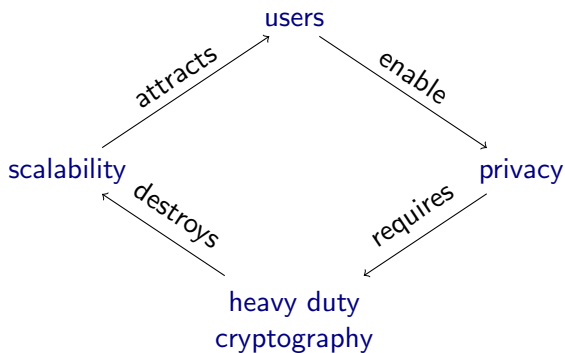
obfuscate origin

π \leftrightarrow

“amounts are positive and
one these UTXOs is the true origin”

Privacy Coins — Disadvantage

Privacy Coins — Disadvantage



Scalability – Four Types of Information

Scalability – Four Types of Information



unverified witness



state update



state information



verified witness

Scalability – Four Types of Information



unverified witness



state update



state information




verified witness




utxo

Scalability – Four Types of Information

 unverified witness

 state update

 state information


 verified witness



utxo


new tx

Scalability – Four Types of Information

 unverified witness

 state update


 state information


 verified witness



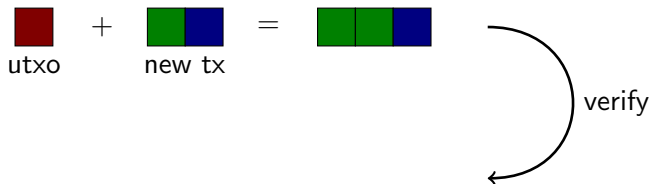
Scalability – Four Types of Information

 unverified witness

 state update


 state information


 verified witness



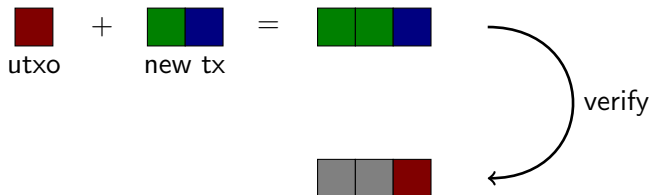
Scalability – Four Types of Information

 unverified witness

 state update

 state information

 verified witness



Scalability – Synking

Scalability – Synking

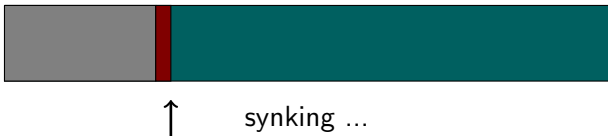
full node:



initially

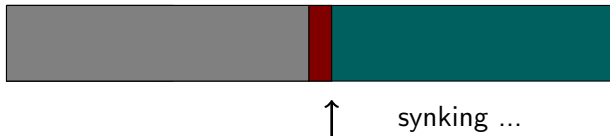
Scalability – Synking

full node:



Scalability – Synking

full node:



Scalability – Synking

full node:



Scalability – Synking

full node:



synked.



Scalability – Synking

full node:



synked.

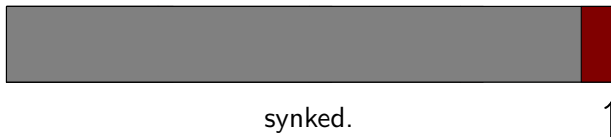


SNARK node:




Scalability – Synking

full node:



SNARK node:

π 
initially

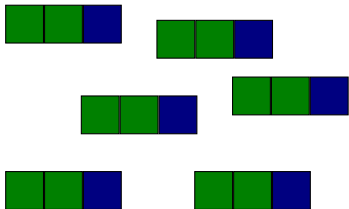
π 
synked.

Scalability — Aggregation

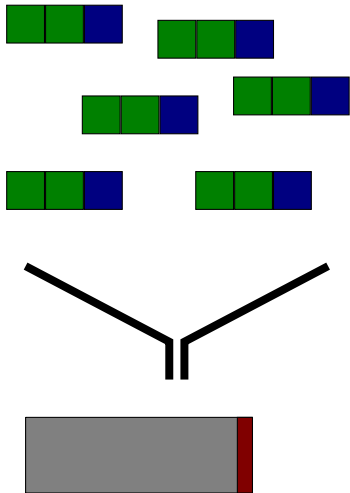
Scalability — Aggregation



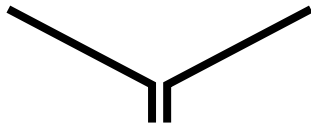
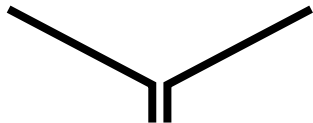
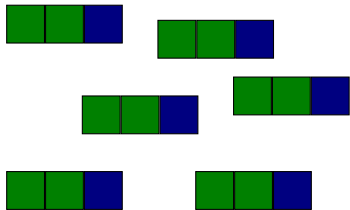
Scalability — Aggregation



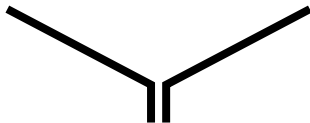
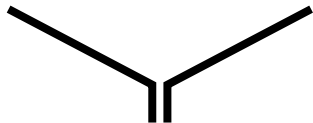
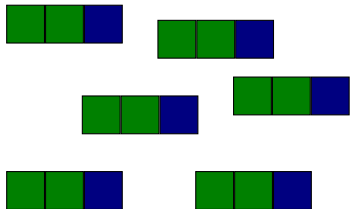
Scalability — Aggregation



Scalability — Aggregation



Scalability — Aggregation



III Applications

III Applications B) Other

Private Finance

Private Finance

trades

guarantee: integral order matching

hide: algorithm, volume

Private Finance

trades

guarantee: integral order matching

hide: algorithm, volume

lending

guarantee: cash flow positivity,
qualification

hide: transaction history

Private Finance

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guarantee: integral order matching

hide: algorithm, volume

investment

guarantee: blacklist non-membership

hide: investor identity, source of funds,
allocation

lending

guarantee: cash flow positivity,
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Private Finance

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guarantee: integral order matching

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guarantee: blacklist non-membership

hide: investor identity, source of funds,
allocation

lending

guarantee: cash flow positivity,
qualification

hide: transaction history

audit

guarantee: regulatory compliance,
valid accounting

hide: balance sheet

Private eGovernment

Private eGovernment

identity

guarantee: qualification

hide: personal data

Private eGovernment

identity

guarantee: qualification

hide: personal data

voting

guarantee: election integrity

hide: individual votes

Private eGovernment

identity

guarantee: qualification

hide: personal data

voting

guarantee: election integrity

hide: individual votes

taxes

guarantee: tax compliance

hide: wealth, assets

Private eGovernment

identity

guarantee: qualification

hide: personal data

voting

guarantee: election integrity

hide: individual votes

taxes

guarantee: tax compliance

hide: wealth, assets

surveillance

guarantee: legality,

blacklist non-membership

hide: movements, interactions,
communications

Other

Other

gaming

guarantee: unbiased shuffling, legal moves

hide: strategy, position, randomness

Other

gaming

guarantee: unbiased shuffling, legal moves

hide: strategy, position, randomness

exploits and vulnerabilities

guarantee: existence of vulnerability, fair exchange

hide: vulnerability