KU LEUVEN

A Practical Multivariate Blind Signature Scheme

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1. Blind Signatures
2. MQ Signatures
   - Rainbow
   - MQDSS
3. Multivariate Blind Signature Scheme
   - Scheme
   - Numbers

Blind Signature

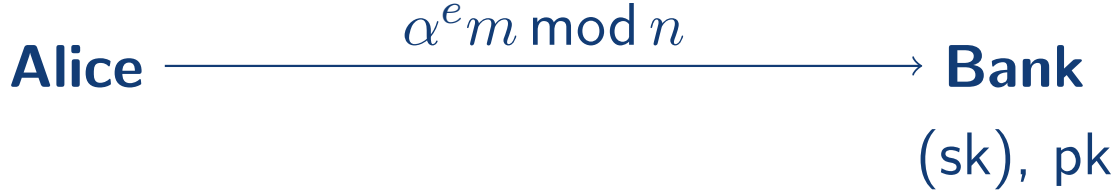
Alice

Bank

(sk), pk

Merchant

Blind Signature



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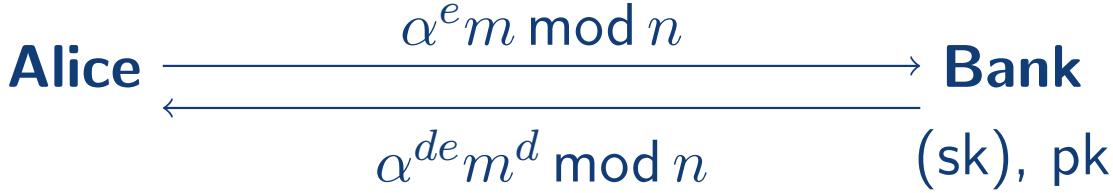

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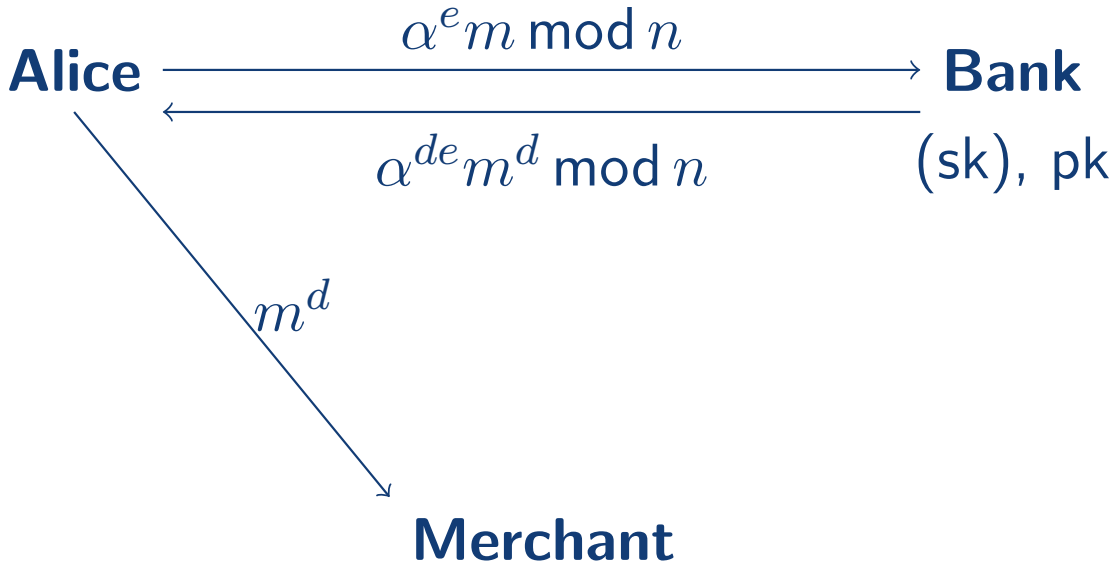

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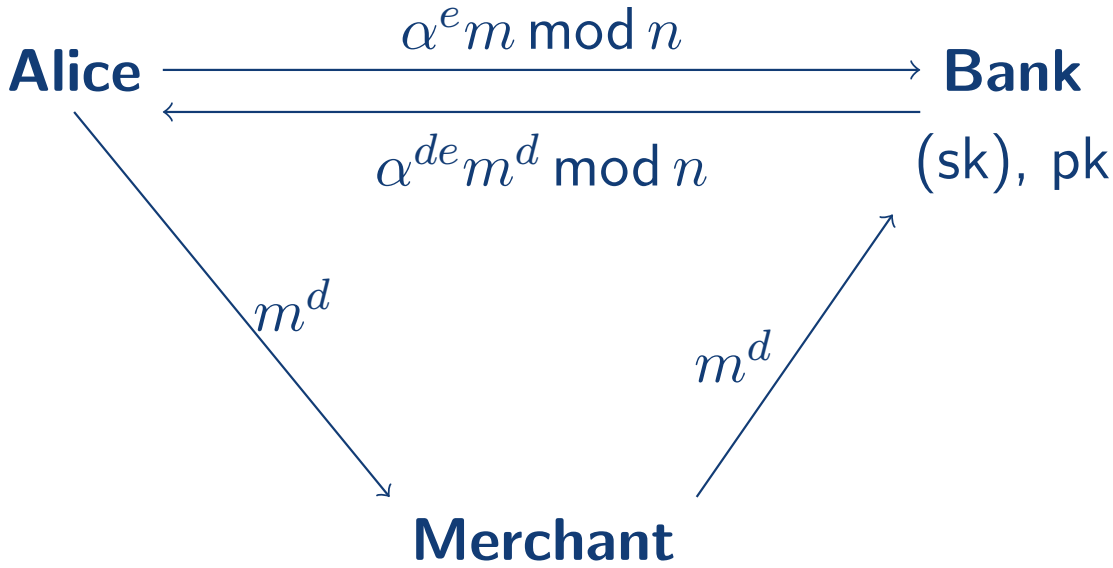
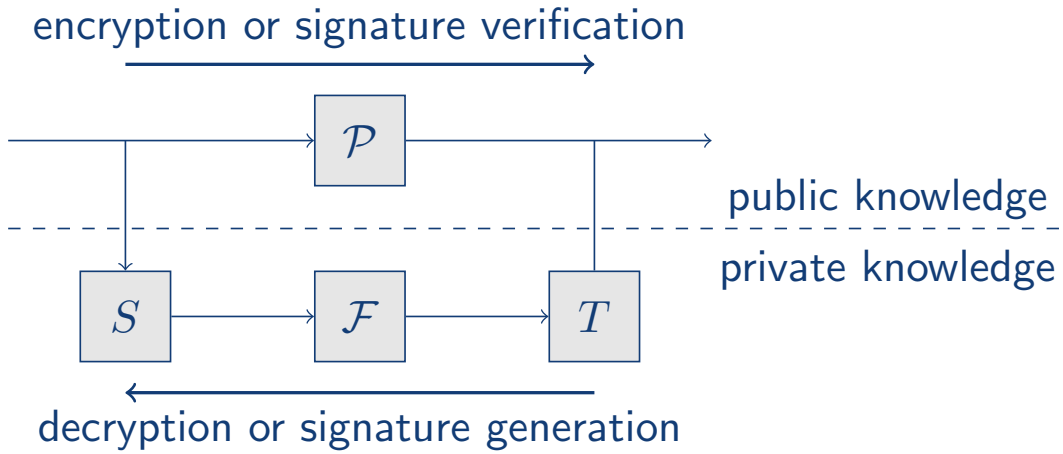


MQ Signature Scheme

- EIP-based
  - HFEv-, UOV, **Rainbow**
  - $\mathcal{P} = T \circ \mathcal{F} \circ S$
  - verify $\mathbf{s}$: $\mathcal{P}(\mathbf{s}) \stackrel{?}{=} \mathcal{H}(m)$



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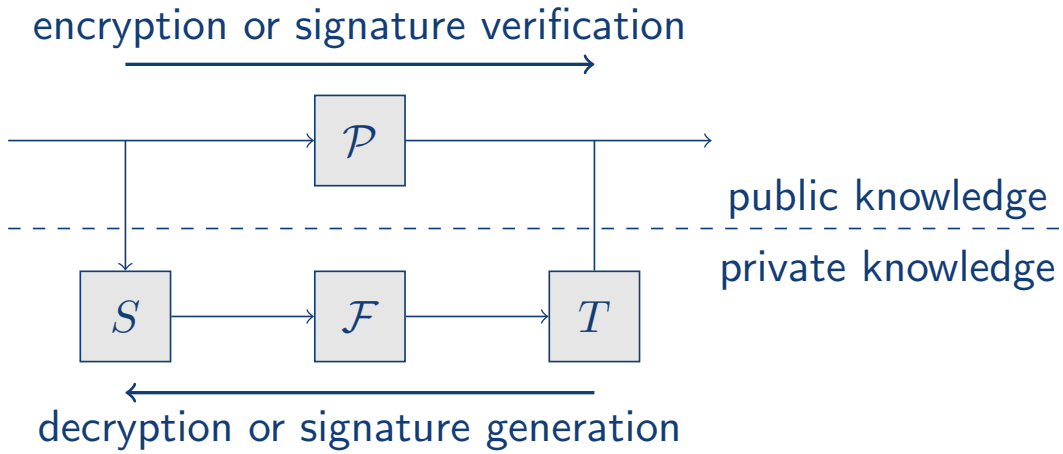

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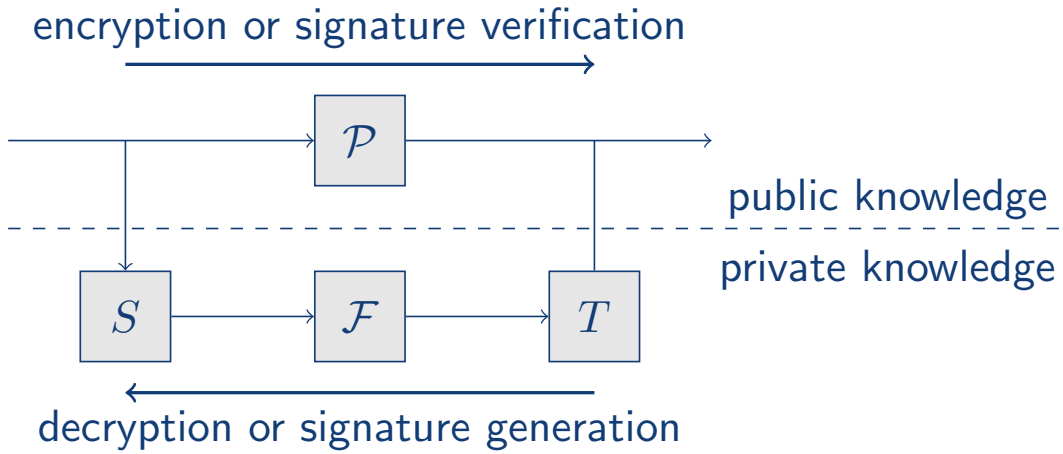

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- Blind Signature: EIP + ZKPoK

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- Unbalanced Oil and Vinegar: precursor to Rainbow
- v *vinegar* variables and o *oil* variables ($v \approx 2o$)
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- signature generation:
  - choose $\mathbf{x}_v \xleftarrow{\$} \mathbb{F}_q^v$
  - solve linear system to obtain $\mathbf{x}_o$ (#eqns = #vars = o)
  - invert linear transformation S

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- signature generation:
  - invert linear transformation T
  - choose $\mathbf{x}_v \xleftarrow{\$} \mathbb{F}_q^v$
  - solve o_1 linear equations to obtain $\mathbf{x}_{o_1}$
  - treat $(\mathbf{x}_v; \mathbf{x}_{o_1})$ as vinegar variables
  - solve o_2 linear equations to obtain $\mathbf{x}_{o_2}$
  - invert linear transformation S

SSH Protocol

- ZKPoK$\{(\mathbf{s}) : \mathcal{P}(\mathbf{s}) = \mathbf{v}\}$
- uses *polar form*: $\mathcal{G}(\mathbf{x}, \mathbf{y}) = \mathcal{P}(\mathbf{x} + \mathbf{y}) - \mathcal{P}(\mathbf{x}) - \mathcal{P}(\mathbf{y}) + \mathcal{P}(\mathbf{0})$

Prover: $\mathcal{P}, \mathbf{s}, \mathbf{v}$		Verifier: $\mathcal{P}, \mathbf{v}$
$\mathbf{r}_0, \mathbf{t}_0 \xleftarrow{\$} \mathbb{F}_q^n; \mathbf{e}_0 \xleftarrow{\$} \mathbb{F}_q^m; \mathbf{r}_1 \leftarrow \mathbf{s} - \mathbf{r}_0$		
$c_0 = Com(\mathbf{r}_0, \mathbf{t}_0, \mathbf{e}_0)$		
$c_1 = Com(\mathbf{r}_1, \mathcal{G}(\mathbf{t}_0, \mathbf{r}_1) + \mathbf{e}_0)$		
	$\xrightarrow{c_0, c_1}$	
	$\xleftarrow{\alpha}$	$\alpha \xleftarrow{\$} \mathbb{F}_q$
$\mathbf{t}_1 \leftarrow \alpha\mathbf{r}_0 - \mathbf{t}_0$		
$\mathbf{e}_1 \leftarrow \alpha\mathcal{P}(\mathbf{r}_0) - \mathbf{e}_0$		
	$\xrightarrow{\mathbf{t}_1, \mathbf{e}_1}$	
	$\xleftarrow{ch}$	$ch \xleftarrow{\$} \{0, 1\}$
	$\xrightarrow{\mathbf{r}_{ch}}$	
		$ch = 0 \rightarrow c_0 \stackrel{?}{=} Com(\cdot)$
		$ch = 1 \rightarrow c_1 \stackrel{?}{=} Com(\cdot)$

MQDSS

- turns SSH protocol into signature scheme
- non-interactive using Fiat-Shamir (sort of)
- optimization for speed and size
- 2.43 ms for signature generation (256 bits security)

Blind Signature Scheme: General Idea

dedicated signature scheme

$+$ basic algebraic properties

$+$ zero-knowledge proof

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Multivariate Blind Signature

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$\mathsf{pk} = (\mathcal{P}, \mathcal{R})$

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$$\mathsf{NIZKPoK}\{(\mathbf{z}, \mathbf{z}^*) \ : \ \mathcal{P}(\mathbf{z}^*) + \mathcal{R}(\mathbf{z}) = \mathcal{H}(m)\}$$

Multivariate Blind Signature

$\mathbf{z} \xleftarrow{\$} \mathbb{F}_q^n$

$\mathbf{w}^* = \mathcal{H}(m) - \mathcal{R}(\mathbf{z})$

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Alice $\xrightarrow{\quad \mathbf{w}^* \quad}$ **Bank**

$\mathbf{z}^*$ st. $\mathcal{P}(\mathbf{z}^*) = \mathbf{w}^*$ $\xleftarrow{\quad}$

$(\mathsf{sk} = (\mathcal{T}, \mathcal{F}, \mathcal{S}))$

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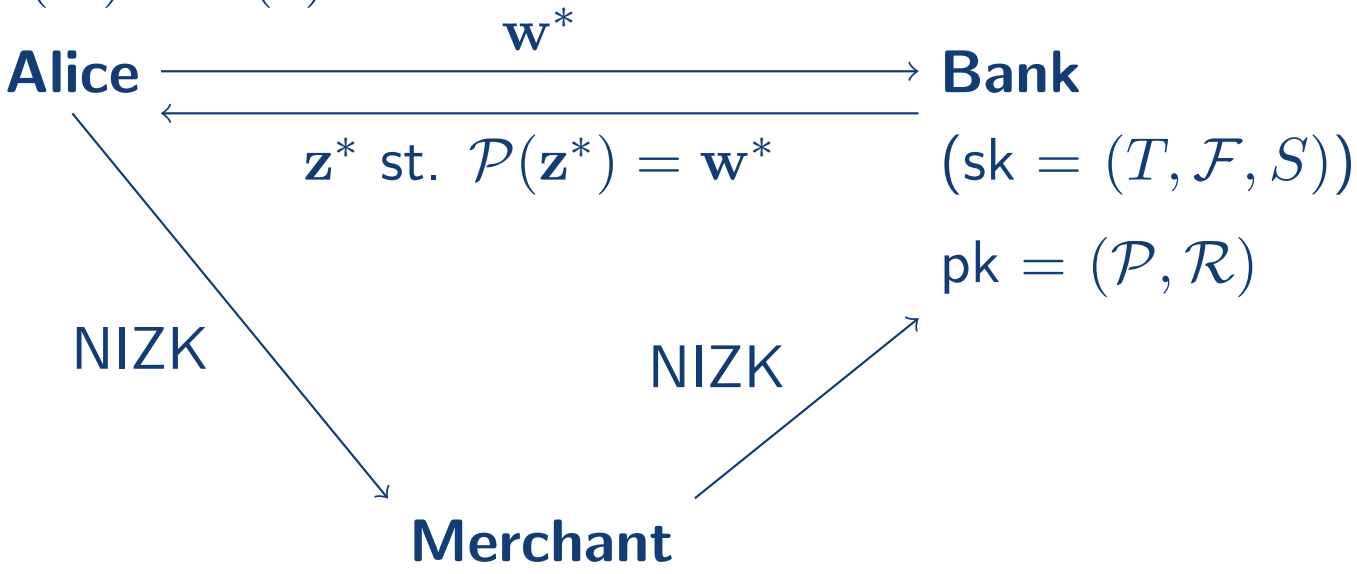

NIZKPoK$\{(\mathbf{z}, \mathbf{z}^*) \ : \ \mathcal{P}(\mathbf{z}^*) + \mathcal{R}(\mathbf{z}) = \mathcal{H}(m)\}$

Security Quirks

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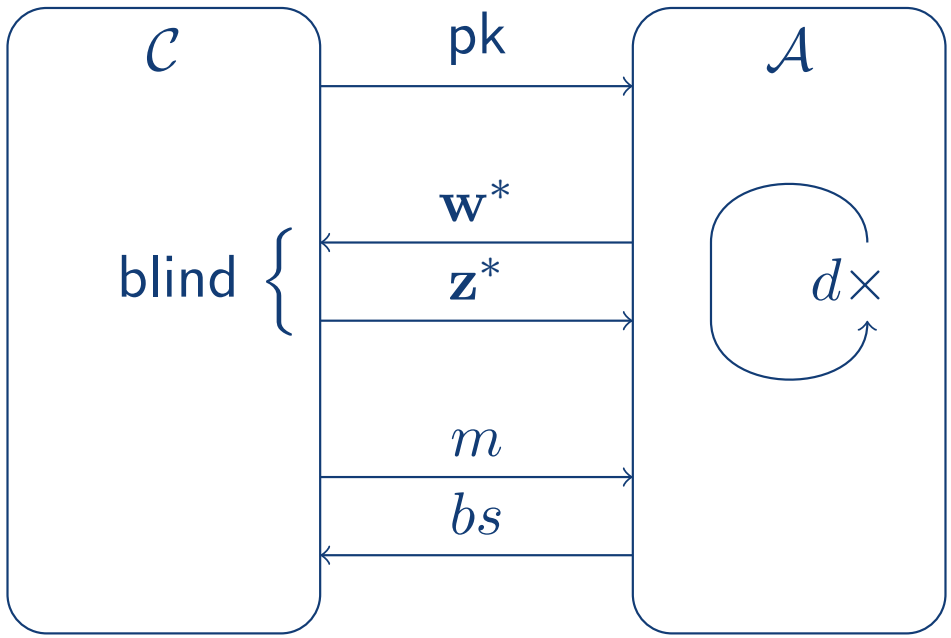
- need *perfectly hiding* commitments for blindness
- *classical* random oracle model
- *universal* one-more unforgeability
  - generalization of UUF-CMA to one-more-unforgeability



parameters, comparison

security level (bit)	parameters $(\mathbb{F}, (v_1, o_1, o_2))$	# rounds	public key size (kB)	private key size (kB)	blind sig. size (kB)
80	(GF(31),(16,18,17))	84	29.4	20.1	11.5
100	(GF(31),(20,22,21))	105	54.6	36.6	17.6
128	(GF(31),(25,27,27))	135	106.8	70.2	28.5
192	(GF(31),(37,35,35))	202	342.8	219.0	63.2
256	(GF(31),(50,53,53))	269	802.4	507.1	111.9

Table: Proposed parameters for our blind signature scheme (GF(31)).

Security lvl. (bit)	Scheme	comm.	Pub. key size (kB)	Sig. size (kB)	Post-quantum?
76	RSA-1229	2	1.2	1.2	×
	Lattice-1024	4	10.2	66.9	✓
	Our scheme	2	29.4	11.5	✓
102	RSA-3313	2	3.3	3.3	×
	Lattice-2048	4	23.6	89.4	✓
	Our scheme	2	54.6	17.6	✓

Table: Comparison of blind signature schemes — RSA / Rückert / ours

Sage implementation

sec. lvl.	Key Gen.	Sign (`Signer`)	Sig. Gen. (`User`)	Sig. Verification
80	4,007	7	2,018	1,424
100	9,392	13	3,649	2,656
128	25,517	19	7,760	5,505
192	87,073	41	23,692	16,040
256	613,968	103	86,540	59,669

Table: Operational speed (milliseconds)