DEEP Commitments

and their Applications

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STARK Workflow with DEEP ALI

AET

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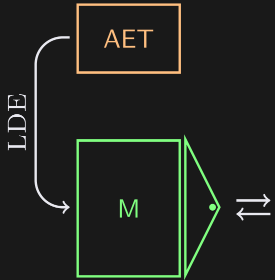

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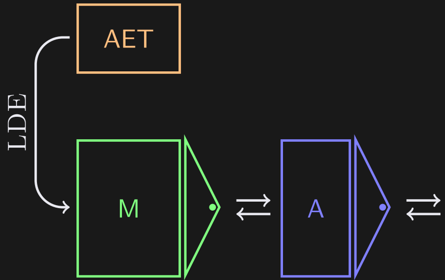

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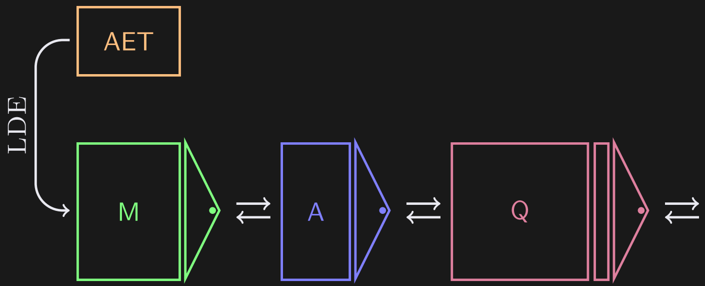

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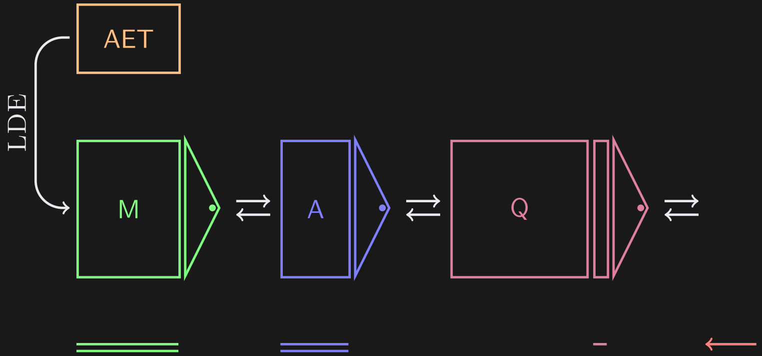

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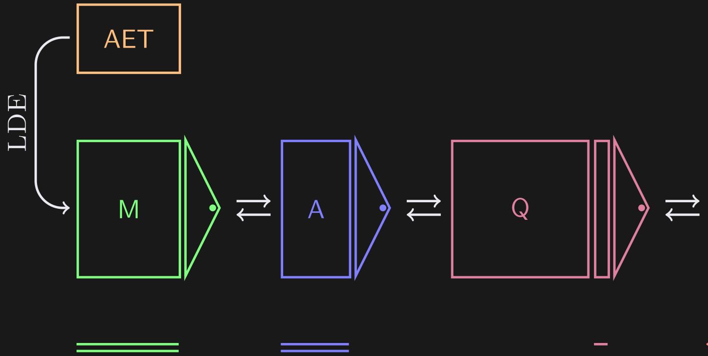
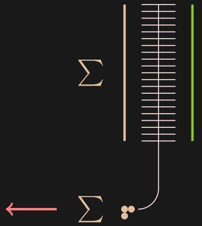


DEEP updates:

$$f^*(X) = \frac{f(X) - f(z)}{X - z}$$



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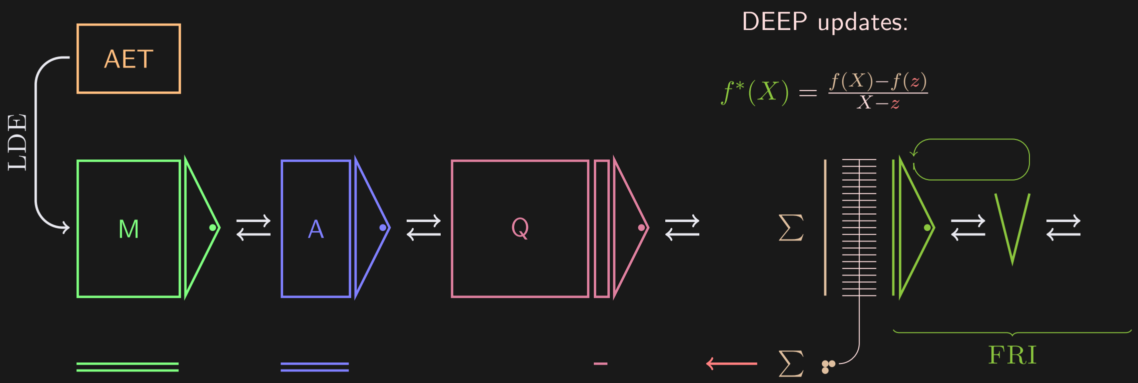

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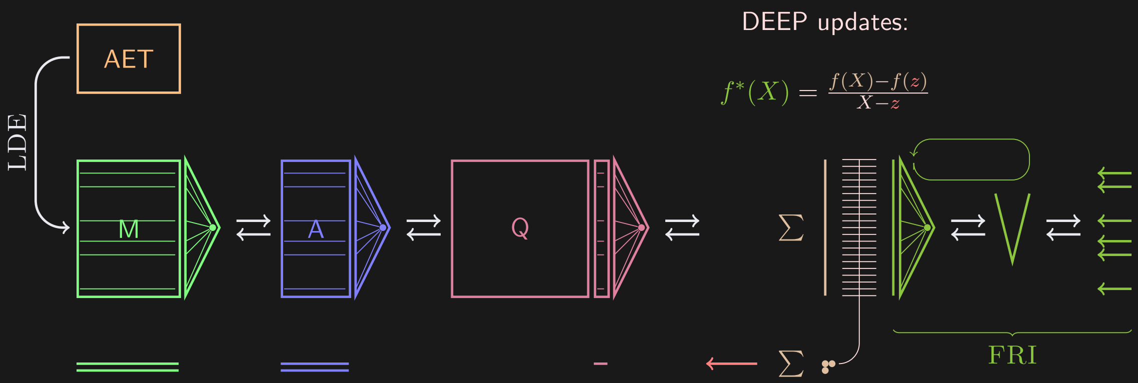
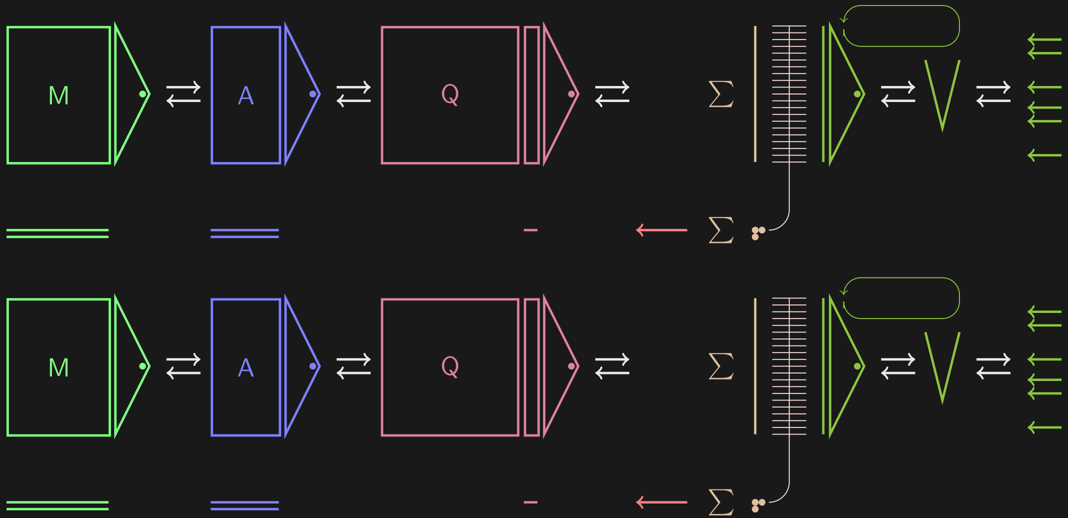
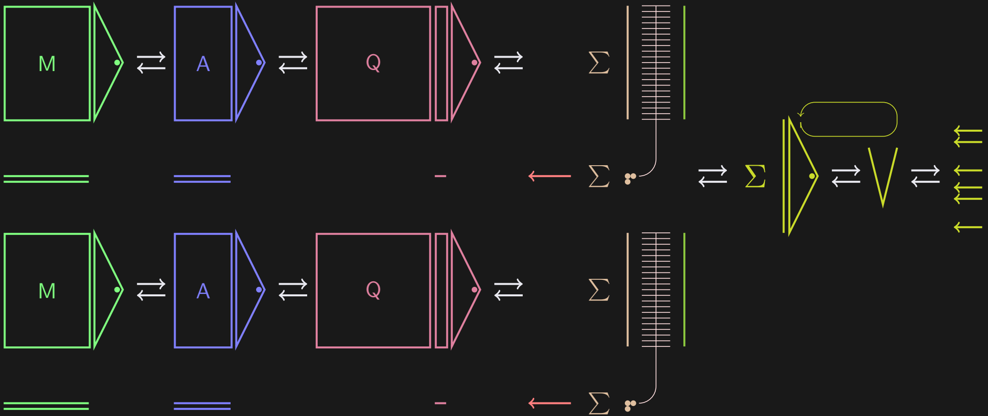


Challenge



Hope: Amortize FRI



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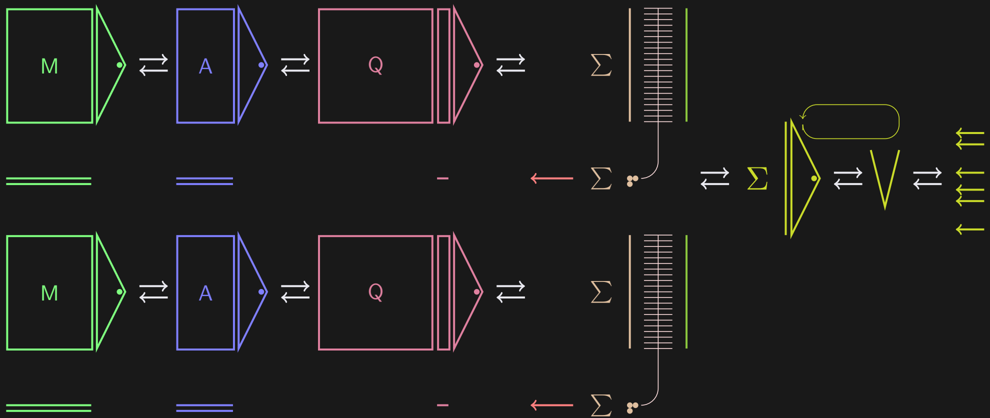
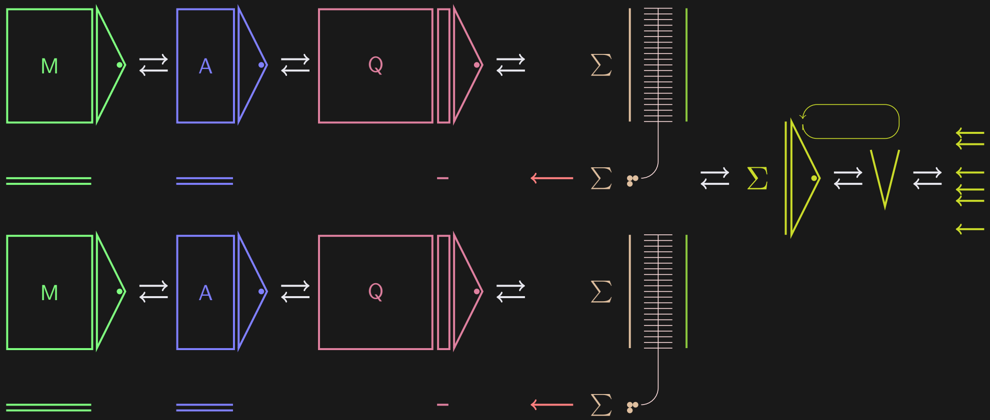


Obstacle 1:

Obstacle 2:

Hope: Amortize FRI



Obstacle 1:

one index set dependent on *both* transcripts

$\Longrightarrow$ prove simultaneously

Obstacle 2:

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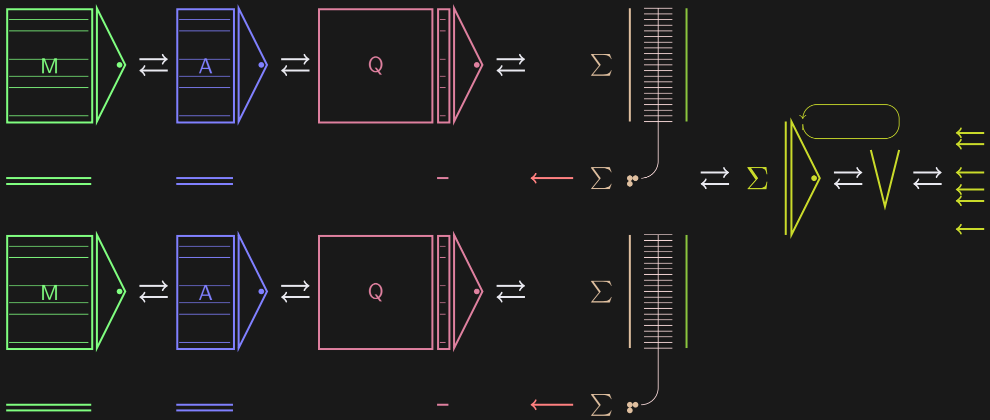

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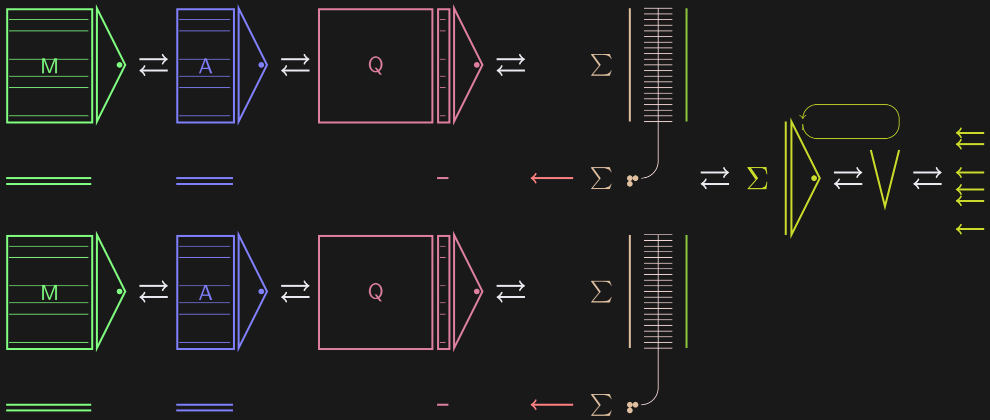

Obstacle 1:

one index set dependent on *both* transcripts

$\Longrightarrow$ prove simultaneously

Obstacle 2:

store *two* AETs

$\longrightarrow$ prohibitive RAM cost

Obstacle 1

$\rightarrow$ split protocol into two parts:

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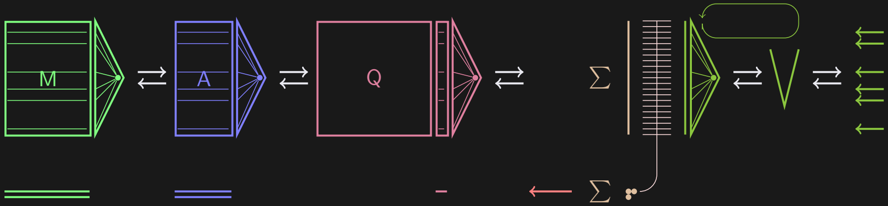

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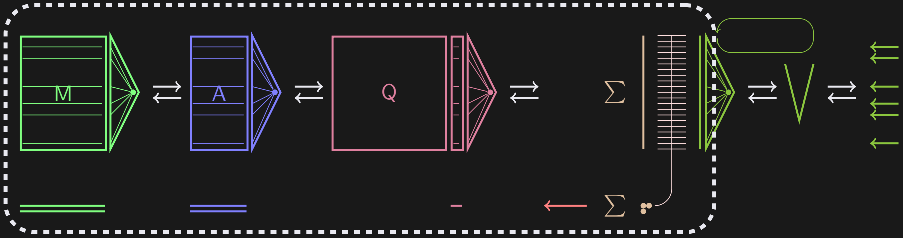

DEEP-ALI: reduce AET $\mapsto$ Merkle root

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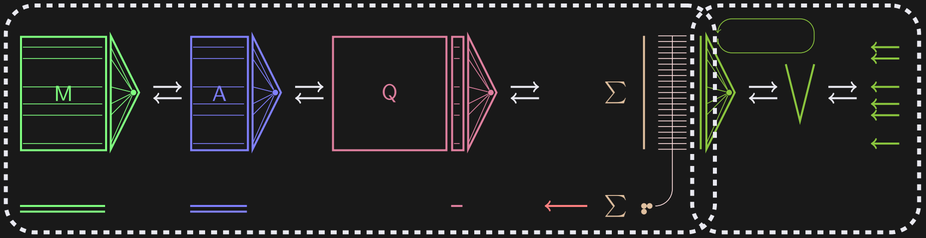

DEEP-ALI: reduce AET ↦ Merkle root

FRI: LDT

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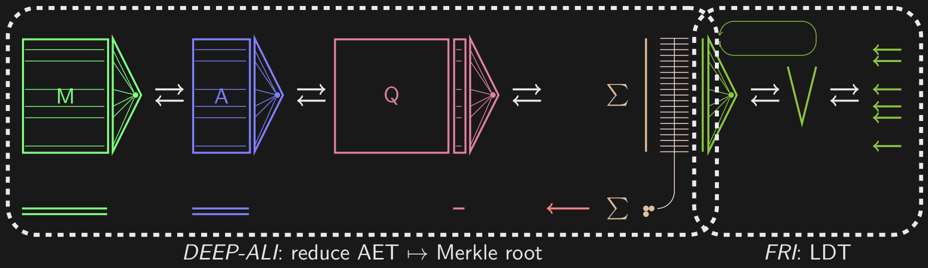

- reduce many AETs to Merkle roots
- concatenate and hash to sample *weights* and *index set*
- run one instance of FRI

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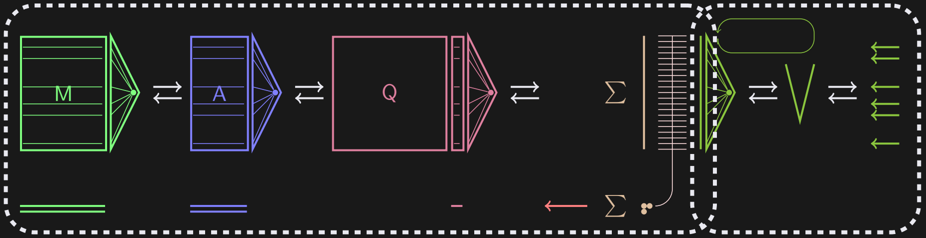

DEEP-ALI: reduce AET ↦ Merkle root

FRI: LDT

– reduce many AETs to Merkle roots ← *any order* ✓

– concatenate and hash to sample *weights* and *index set*

– run one instance of FRI ← "simultaneous"

RAM cost remains! × × × ×

Obstacle 2

$\rightarrow$ decouple codeword from history

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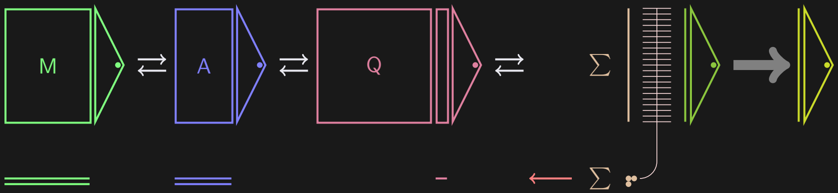
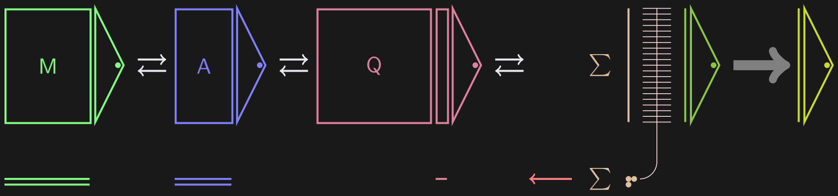


Obstacle 2

$\rightarrow$ decouple codeword from history



- verify authentic reduction to history-independent codeword
- drop AET from RAM
- batch history-independent codewords for FRI

Obstacle 2

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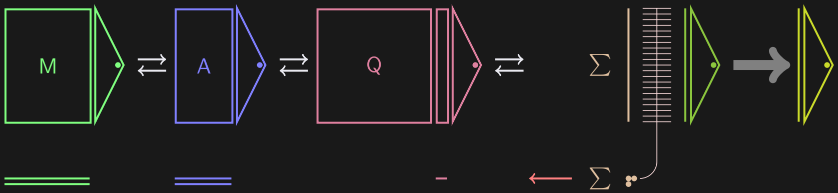

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HOW?

Decouple

$\mathcal{P}$ $h(X)$

$\mathcal{V}$ $[h(X)]$

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$z \xleftarrow{\$} \mathbb{F} \backslash D$

$\xleftarrow{\quad z \quad}$

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$y = h(\alpha)$

$\xrightarrow{\quad y \quad}$

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$x_0, \ldots, x_{t-1} \xleftarrow{\$} D$

$\gamma \xleftarrow{\$} \mathbb{F}$

$\xleftarrow{\quad x_0, \ldots, x_{t-1}, \gamma \quad}$

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$$q(X) = \underbrace{\frac{\gamma^{t+1} \cdot X^{t+1} - 1}{\gamma \cdot X - 1}}_{\text{degree-correction}} \cdot \frac{h(X) - \mathsf{Ans}(X)}{Z(X)}$$

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$$[q(X)] = \begin{cases} \frac{\gamma^{t+1} \cdot X^{t+1} - 1}{\gamma \cdot X - 1} \cdot \frac{[h(X)] - \text{Ans}(X)}{Z(X)} & \Leftarrow X \notin \{x_i\} \\ [\text{Fill}(X)] & \Leftarrow X \in \{x_i\} \end{cases}$$

Soundness

$\sum f_j(X)$

true batch sum

$h(X)$

claimed batch sum

$q(X)$

result

$\Delta(\sum f_j(X), \mathsf{RS}) > \delta$

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Soundness

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true batch sum | claimed batch sum | result

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$\Delta\big(h(X), \mathsf{RS}\big) > \delta$ $\leftarrow$ trivial: by construction up to t points from $\mathsf{Fill}(X)$

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s.t. $\Delta\left(h(X), p(X)\right) < \delta$

and $p(z) = y$

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$h(X)$ — claimed batch sum

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$\nexists\, p(X)$ ← trivial: quotient by wrong interpolant

$\exists\, p(X)$ ← then:

- $p(X)$ is unique whp (DEEP technique)
- since $\Delta(\sum f_i(X), p(X)) > \delta$, wrong in-domain points $\{(x_i, \sum f_j(x_i))\}$ whp
- $\Rightarrow$ quotient by wrong interpolant $\square$

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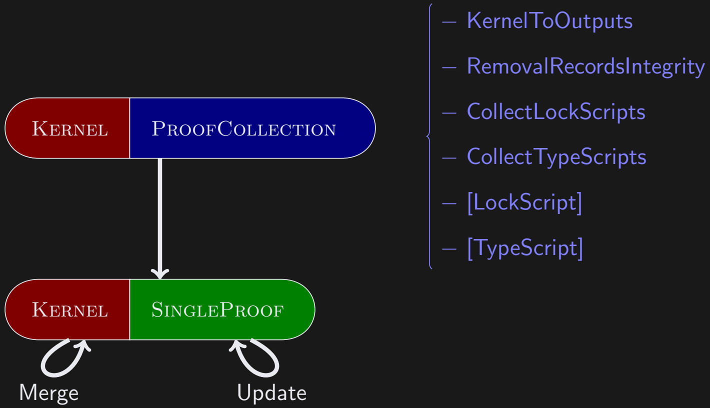

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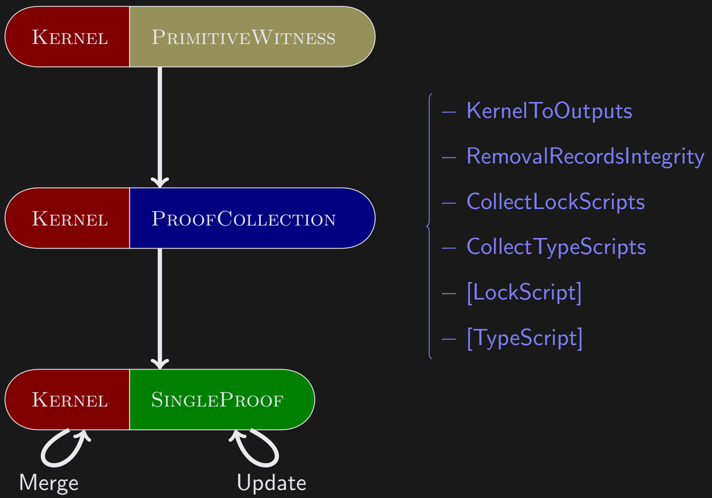

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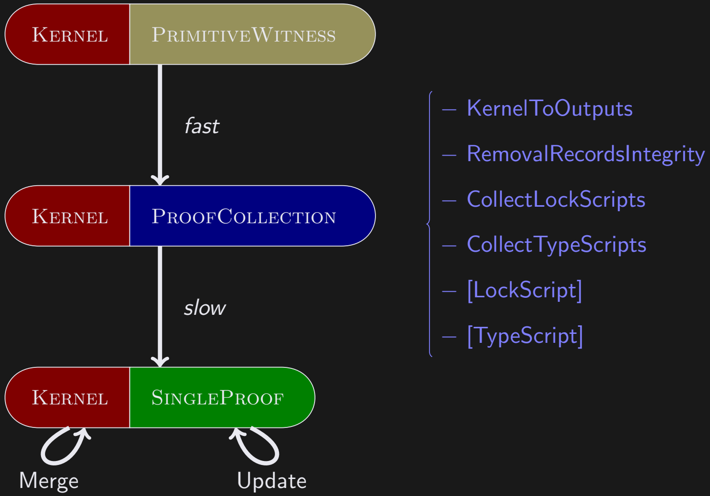

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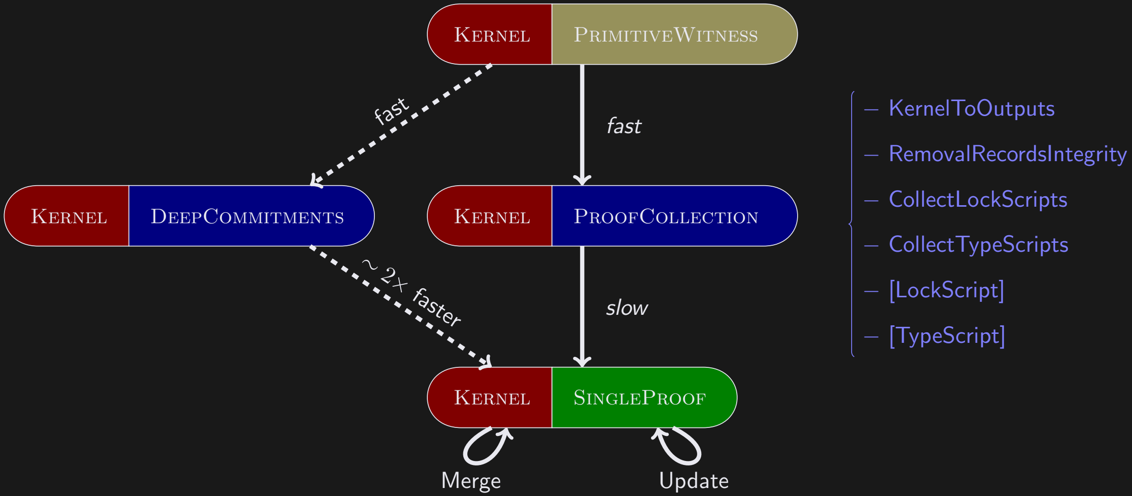

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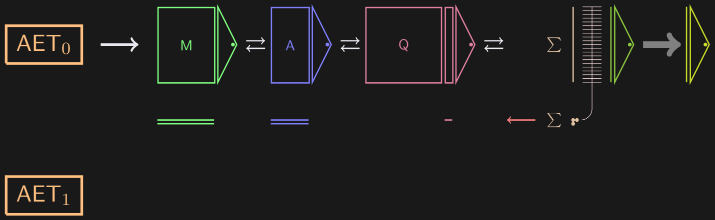

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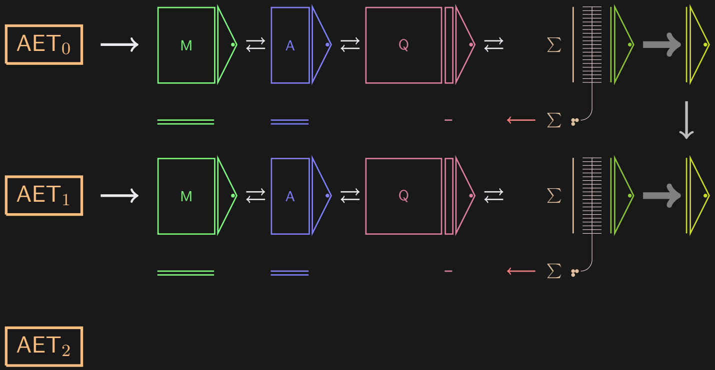

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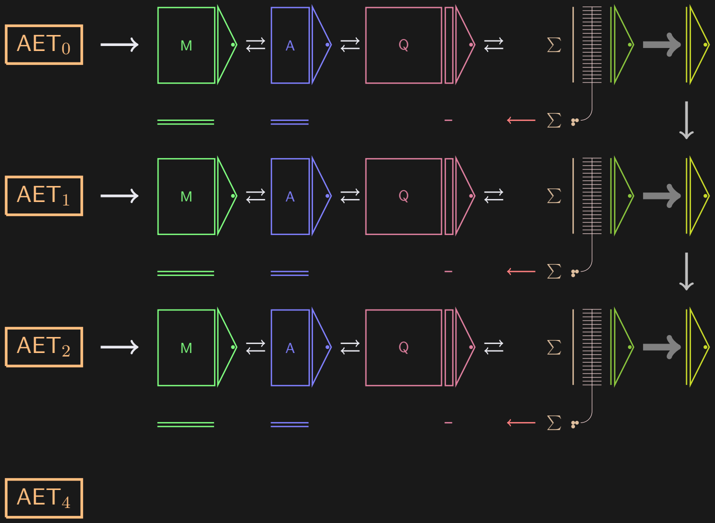

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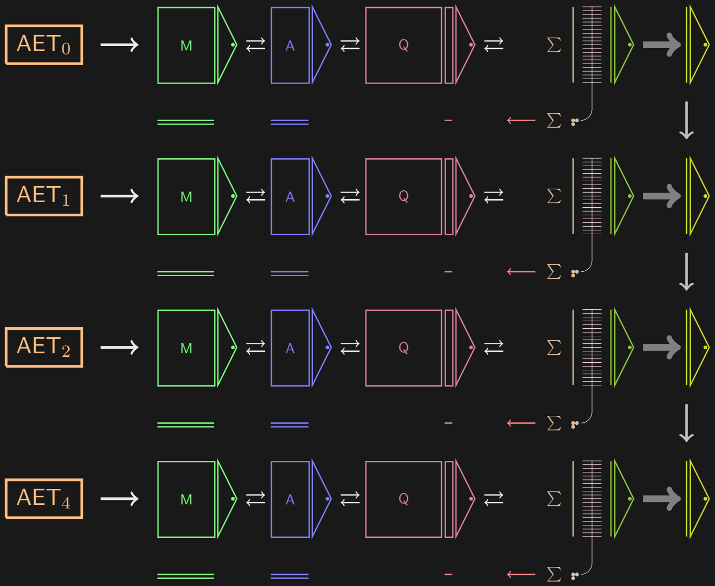

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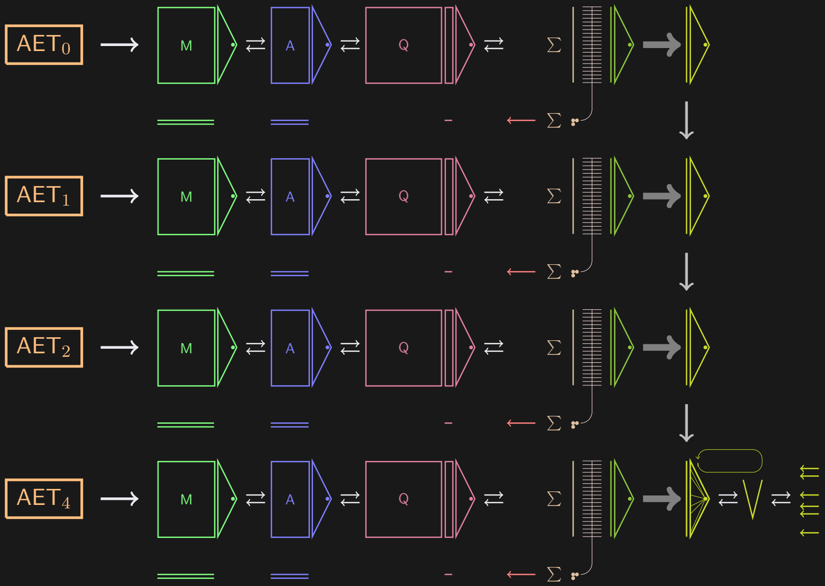

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