Polynomial Acceleration

for STARK VMs

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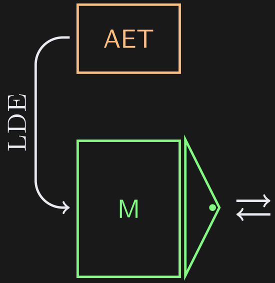
STARK Workflow with DEEP ALI

AET

STARK Workflow with DEEP ALI



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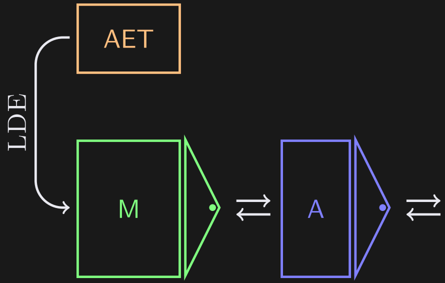

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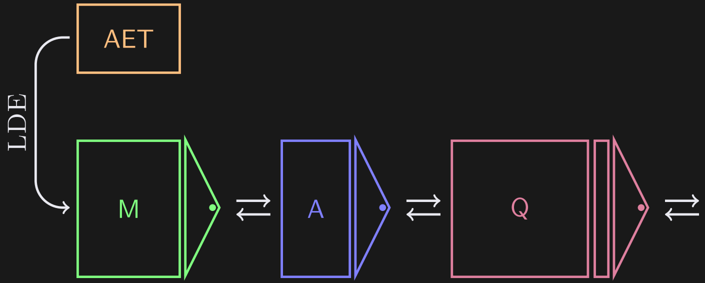

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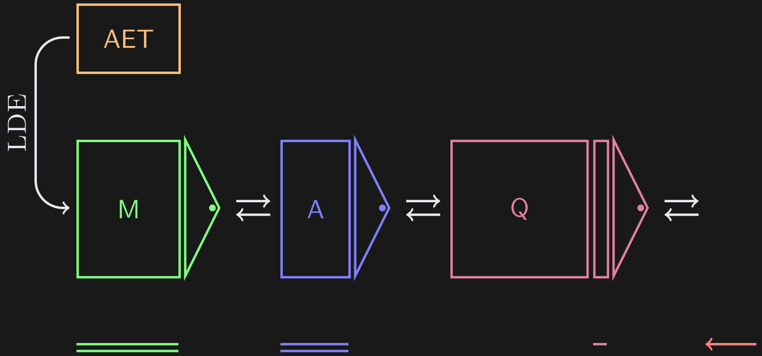

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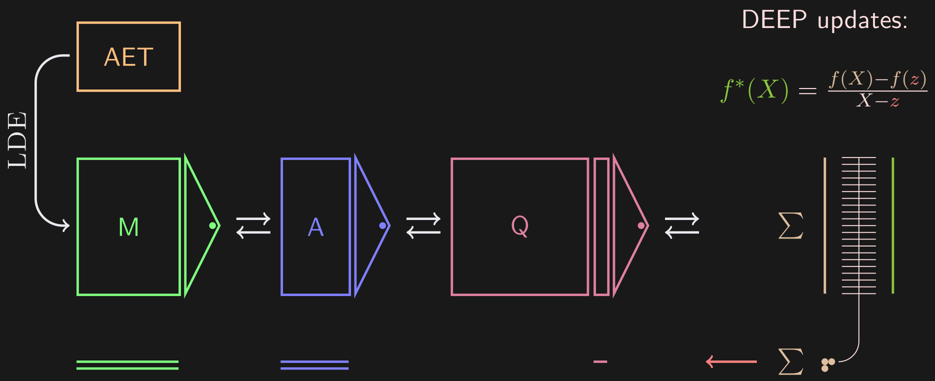

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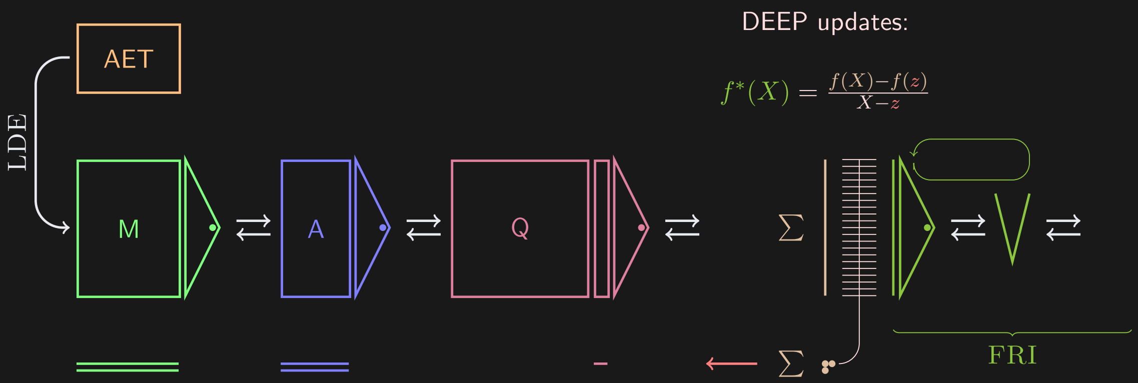

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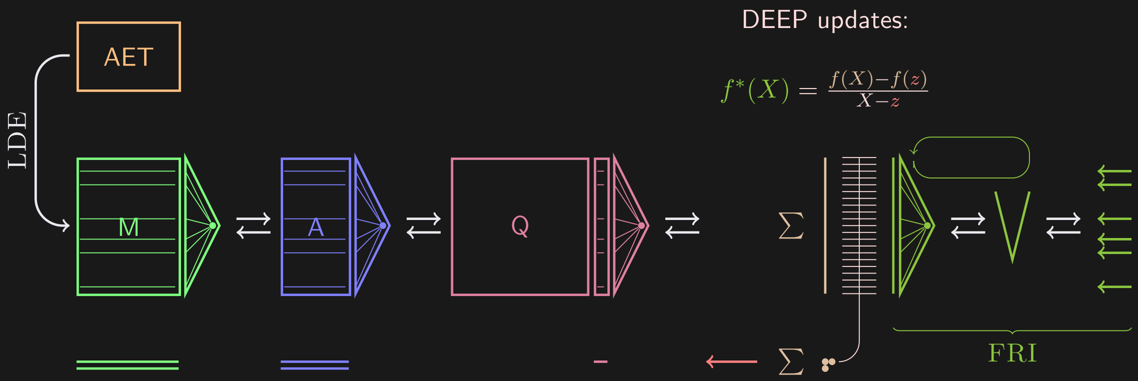
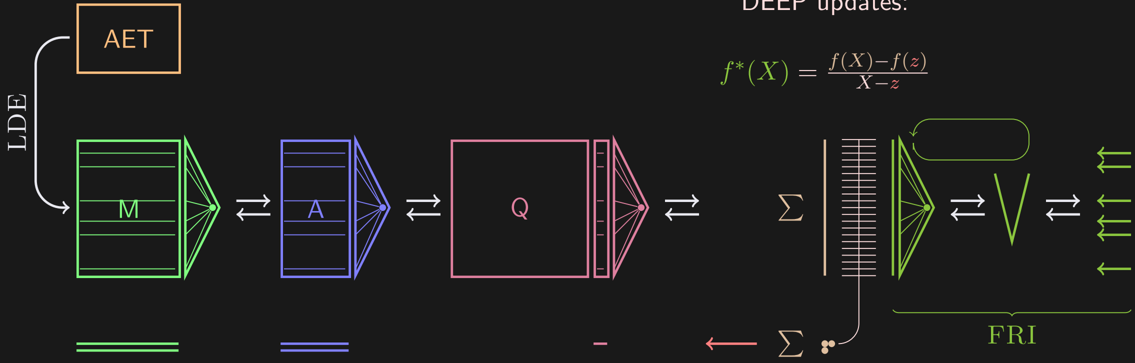


STARK Workflow with DEEP ALI

DEEP updates:

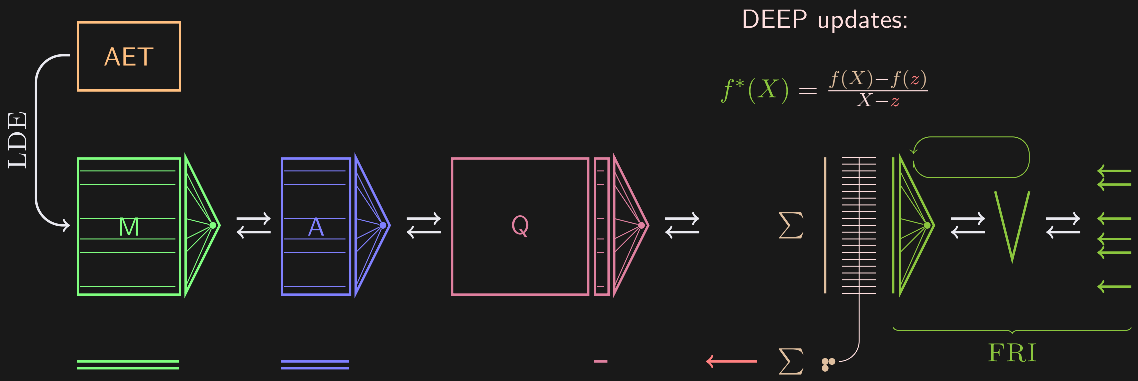
$$f^*(X) = \frac{f(X) - f(z)}{X - z}$$



TWO STAGES:

1. main
2. auxiliary

STARK Workflow with DEEP ALI



TWO STAGES:

1. main
2. auxiliary

Why not < 2?
How about > 2?

Two Stages

Two Stages

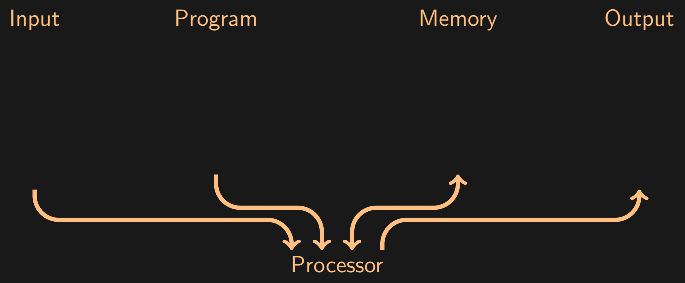
Processor

Two Stages

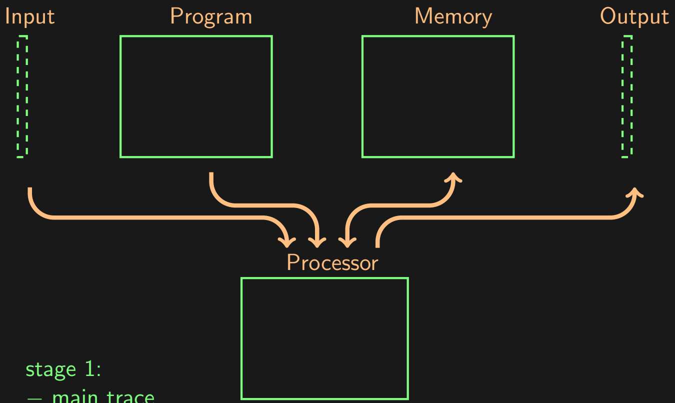
Input Program Memory Output

Processor

Two Stages



Two Stages

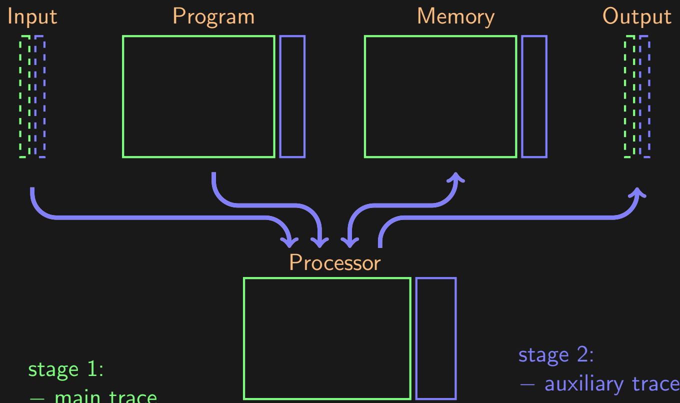


stage 1:

— main trace

— static

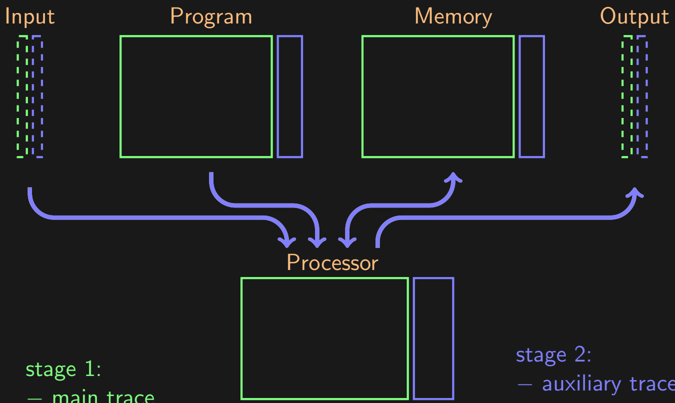
Two Stages



stage 1:
— main trace
— static

stage 2:
— auxiliary trace
— randomized
— cross-table arguments

Two Stages

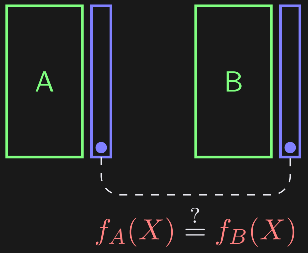


stage 1:
– main trace
– static

stage 2:
– auxiliary trace
– randomized
– cross-table arguments

Auxiliary columns { *link disparate logical units;*
depend on verifier randomness.

Cross-Table Arguments



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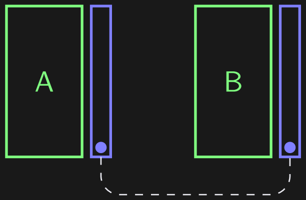

$$f_A(X) \stackrel{?}{=} f_B(X)$$

order / multiplicity	argument	update rule
x / x	*lookup*	$f_A^{(i+1)}(X) = f_A^{(i)}(X) + \frac{\mathsf{mult}_{i+1}}{X - \mathsf{A}_{i+1}}$
x / ✓	*permutation*	$f_A^{(i+1)}(X) = f_A^{(i)}(X) \cdot (X - \mathsf{A}_{i+1})$
✓ / ✓	*evaluation*	$f_A^{(i+1)}(X) = f_A^{(i)}(X) \cdot X + \mathsf{A}_{i+1}$

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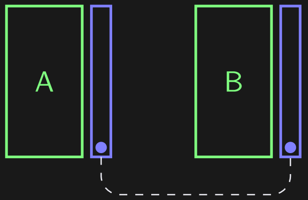

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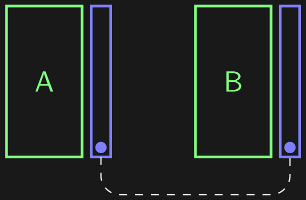

$$f_A(X) \stackrel{?}{=} f_B(X)$$

problem: AIR takes *scalars* not *polynomials*

solution: *collapse* polynomials to scalars

using *evaluation* in $\alpha \stackrel{\$}{\leftarrow} \mathbb{F}$

sound? Yes! Because Schwartz-Zippel.

order / multiplicity	argument	update rule
x / x	*lookup*	$f_A^{(i+1)}(X) = f_A^{(i)}(X) + \frac{\mathsf{mult}_{i+1}}{X - \mathsf{A}_{i+1}}$
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Two Stages

1. Why > 1 stage?

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   - $\rightarrow$ use elementary polynomial arithmetic to link tables
   - $\rightarrow$ collapse polynomials into scalars for AIR
   - $\rightarrow$ using *evaluation* in a *random point* $\alpha \stackrel{\$}{\leftarrow} \mathbb{F}$
   - $\rightarrow$ supplied by the *Verifier*

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- $\rightarrow$ going from 1 stage $\rightarrow$ 2 stages unlocks power
- $\rightarrow$ going from 2 stages $\rightarrow$ 3 stages *unlocks which powers*?

Two Stages

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   $\rightarrow$ use elementary polynomial arithmetic to link tables

   $\rightarrow$ collapse polynomials into scalars for AIR

   $\rightarrow$ using *evaluation* in a *random point* $\alpha \xleftarrow{\$} \mathbb{F}$

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2. How about > 2 stages?

   $\rightarrow$ going from 1 stage $\rightarrow$ 2 stages unlocks power

   $\rightarrow$ going from 2 stages $\rightarrow$ 3 stages *unlocks which powers*?

   $\rightarrow$ *arbitrary* polynomial arithmetic

Virtual Machine State Evolution

traditional | polynomial acceleration | after substitution $X \mapsto \alpha$

instructions:

- basic arithmetic ✓
- add, multiply ✓
- polynomial commitment ✓
- polynomial division + remainder ✓
- polynomial evaluation queries ✓
- degree bound checks ✓

Application 1: Big Integer Arithmetic

$$a \times b$$

$a, b \in \mathbb{N}$ and $a, b \gg p$

1. construct $a(X), b(X), c(X)$ with coefficients $\in \{0, 1\}$
2. calculate $r(X) = (a(X) \cdot b(X) - c(X))\,\%(X - 2)$
3. assert $r(X) = 0$

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cycles:

linear	1. construct $a(X), b(X), c(X)$ with coefficients $\in \{0, 1\}$
constant	2. calculate $r(X) = (a(X) \cdot b(X) - c(X)) \,\%(X - 2)$
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 # cycles:

# cycles	
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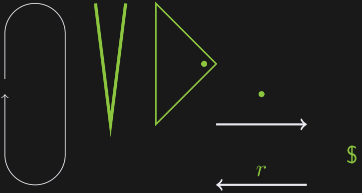
(independent of circuit depth)

$a(X) \cdot b(X) \equiv c(X) \mod X - 2$ (assumes no coefficient overflow)

Application 2: Last Codeword in FRI

many rounds:



last round:

1. Horner evaluation
2. barycentric extrapolation
3. t colinearity checks

polynomial acceleration:

1. construct polynomial
2. evaluate t times
3. t colinearity checks

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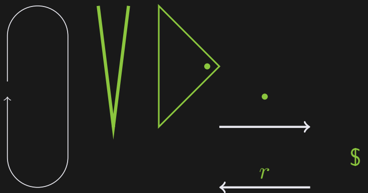

last round:

polynomial acceleration:

	# cycles:
1. Horner evaluation	$O(d)$
2. barycentric extrapolation	$O(\rho^{-1} \cdot d)$
3. t colinearity checks	$O(t)$
1. construct polynomial	$O(d)$
2. evaluate t times	$O(t)$
3. t colinearity checks	$O(t)$

Application 3: SIMD

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$$s \leftarrow \sum_{x \in \langle \omega \rangle} a(x) \quad \Leftrightarrow \quad s \leftarrow \sum_i a_i$$

$\rightarrow$ still need to assert

$$\exists\, c(X) : a(X) - \frac{s}{|\langle \omega \rangle|} = c(X) - c(\omega X)$$

Application 3: SIMD

linear construction $\rightarrow$ $a(X) + b(X) \Leftrightarrow \mathbf{a} + \mathbf{b}$

$a(X) \times b(X) \,\%\, X^N - 1 \Leftrightarrow \mathbf{a} \circ \mathbf{b}$

$a(\omega^i) \Leftrightarrow a_i$

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batchable $\rightarrow$ $\exists\, c(X) : a(X) - \frac{s}{|\langle \omega \rangle|} = c(X) - c(\omega X)$

Construction

pr1	pr1d	···	prn	prnd	r1	r1d	r2	r2d	**acc**	ninv	dinv	qinv	as1	···	as5

Construction

pr1	pr1d	⋯	prn	prnd	r1	r1d	r2	r2d	acc	ninv	dinv	qinv	as1	⋯	as5
$n(X)$	X^{-b_n}		$d(X)$	X^{-b_d}	$q(X)$	X^{-b_q}	$r(X)$	X^{-b_r}	$a(X,Y)$	X^{b_n}	X^{b_d}	X^{b_q}			

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$n(X)$	X^{-b_n}		$d(X)$	X^{-b_d}	$q(X)$	X^{-b_q}	$r(X)$	X^{-b_r}	$a(X,Y)$	X^{b_n}	X^{b_d}	X^{b_q}	$a_{-1}(X,\beta)$		

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$n(X)$	X^{-b_n}		$d(X)$	X^{-b_d}	$q(X)$	X^{-b_q}	$r(X)$	X^{-b_r}	$a(X,Y)$	X^{b_n}	X^{b_d}	X^{b_q}	$a_{-1}(X,\beta)$		
$n(\alpha)$	α^{-b_n}		$d(\alpha)$	α^{-b_d}	$q(\alpha)$	α^{-b_q}	$r(\alpha)$	α^{-b_r}	$a(\alpha,\beta)$	α^{b_n}	α^{b_d}	α^{b_q}			

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div-mod: $q(X) \leftarrow n(X) \,//\, d(X) \quad r(X) \leftarrow n(X) \,\%\, d(X)$

$n(X) = d(X) \cdot q(X) + r(X)$ $\downarrow$ AIR ✓

$\deg(r) < \deg(d)$ $\downarrow$

$\deg(d) + \deg(q) = \deg(n)$ $\downarrow$

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→ AIR ✓

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$\rightarrow$ AIR ✓

$$\deg_X(a_{-1}(X,Y)) \le N \overset{\text{SZL}}{\Leftrightarrow} \deg(a_{-1}(X,\beta)) \le N$$

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FRI: $a_{-1}(X,\beta) \Downarrow \deg(a_{-1}(X,\beta)) \le N$

div-mod: $q(X) \leftarrow n(X) \,//\, d(X)$ $\quad r(X) \leftarrow n(X) \,\%\, d(X)$

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$\rightarrow$ AIR ✓

$\deg(a_{-1}(X,\beta)) \le N$

$$\deg_X(a_{-1}(X,Y)) \le N \overset{\text{SZL}}{\Leftrightarrow} \deg(a_{-1}(X,\beta)) \le N$$

Construction

pr1	pr1d	⋯	prn	prnd	r1	r1d	r2	r2d	acc	ninv	dinv	qinv	as1	⋯	as5
$n(X)$	X^{-b_n}		$d(X)$	X^{-b_d}	$q(X)$	X^{-b_q}	$r(X)$	X^{-b_r}	$a(X,Y)$	X^{b_n}	X^{b_d}	X^{b_q}	$a_{-1}(X,\beta)$		
$n(\alpha)$	α^{-b_n}		$d(\alpha)$	α^{-b_d}	$q(\alpha)$	α^{-b_q}	$r(\alpha)$	α^{-b_r}	$a(\alpha,\beta)$	α^{b_n}	α^{b_d}	α^{b_q}			

FRI ↓

div-mod: $q(X) \leftarrow n(X) \,//\, d(X) \quad r(X) \leftarrow n(X) \,\%\, d(X)$

$n(X) = d(X) \cdot q(X) + r(X)$ ↓ AIR ✓

$\deg(r) < \deg(d)$ ↓

$a \mapsto Y \cdot a + X^N \cdot X^{-b_r} \cdot r(X)$

$a \mapsto Y \cdot a + X^{b_d - b_r - 1}$

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$a \mapsto Y \cdot a + X^{b_q + b_d - b_n}$

→ AIR ✓

$\deg(a_{-1}(X,\beta)) \leq N$

$a(\alpha,\beta)$

$\deg_X(a_{-1}(X,Y)) \leq N \overset{\text{SZL}}{\Leftrightarrow} \deg(a_{-1}(X,\beta)) \leq N$

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$\deg(a_{-1}(X,\beta)) \leq N$

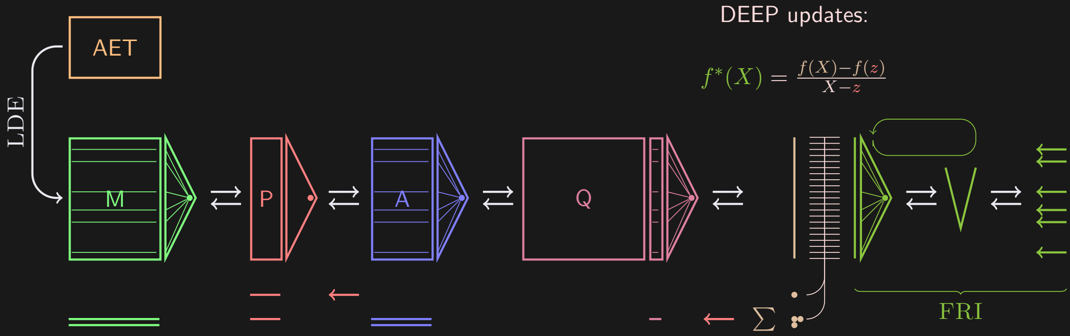
$\updownarrow$

- OODE in α
- DEEP update
- AIR ✓

$a(\alpha,\beta)$

$$\deg_X(a_{-1}(X,Y)) \leq N \overset{\text{SZL}}{\Leftrightarrow} \deg(a_{-1}(X,\beta)) \leq N$$

STARK Workflow with DEEP ALI and Polynomial Acceleration



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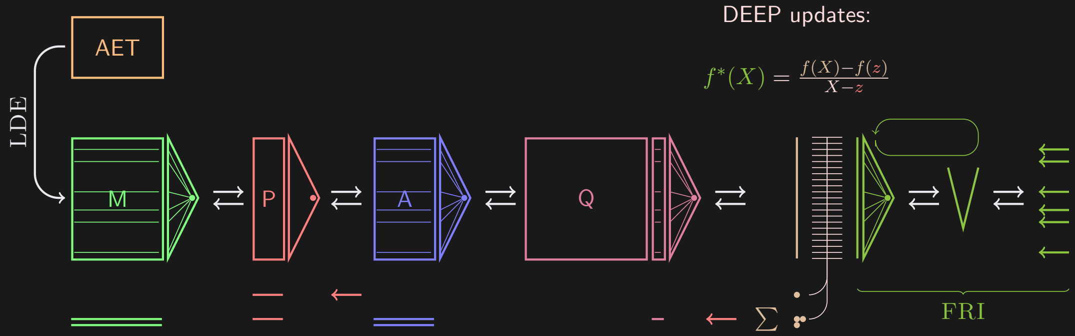

THREE STAGES:

1. main
2. accumulator polynomial segments
3. auxiliary

Limitations

1. No Composition
   - AIR applies to *scalars*
   - no scaling or sign flips
   - dihedral permutations *or* SIMD multiply
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   - AIR applies to *scalars*
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   - *can* verify compositions but ...

2. No Divination
   - no non-determinism
   - VM runs on *prover-side* not *verifier-side*
   - decommitment takes *linear* # AET cells

Open Problem

Sublinear Construction

Open Problem

Sublinear Construction

– verify that a *divined* polynomial of degree N

corresponds uniquely* to a constant-size cryptographic digest

in $< O(N)$ cycles

using polynomial acceleration instructions (including SIMD)

*: up to cryptographic hardness assumptions

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Achieves:

- simulate arbitrary Polynomial IOPs *as prover*

  $\rightarrow$ shifts cost to *outside of AET*