The Tip5 Hash Function for Recursive STARKs

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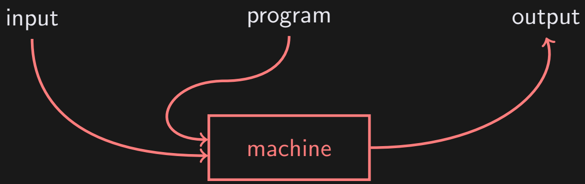
Polygon

neptune

Triton VM

STARK

STARK



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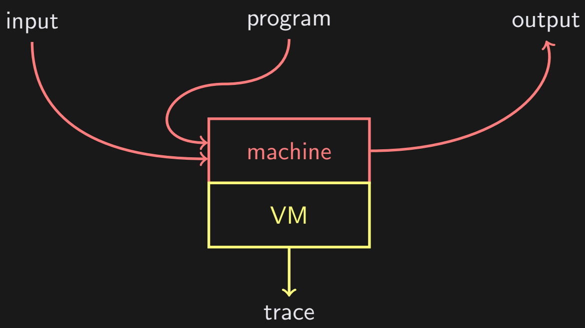
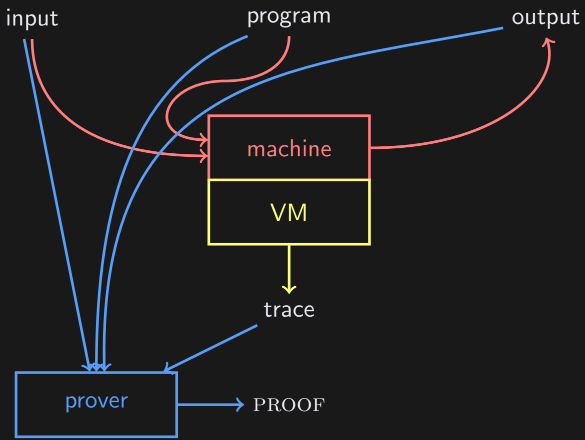


STARK



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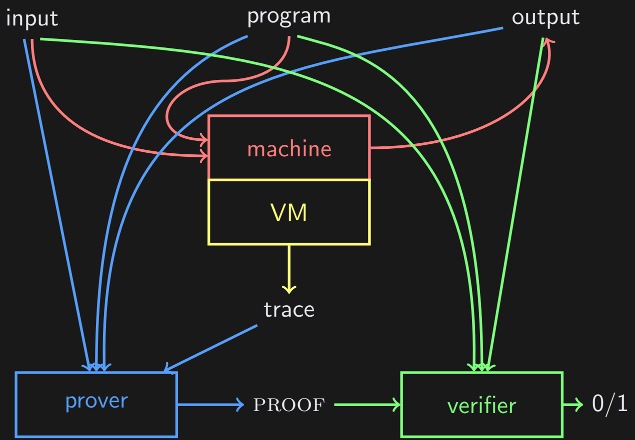
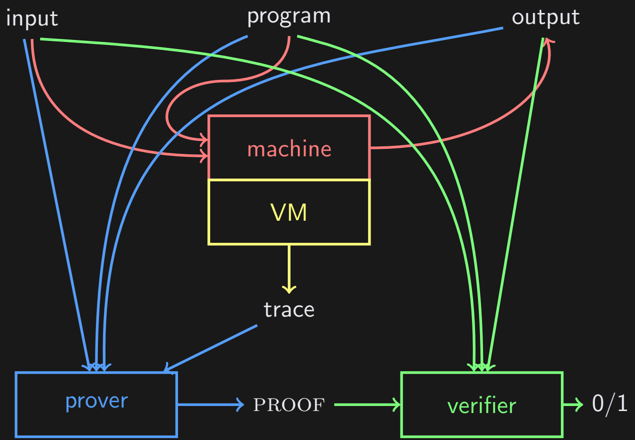

input
program
output
machine
VM
trace
prover
PROOF
verifier
0/1

STARK



✓ fast verifier
✓ transparent
✓ zero-knowledge
✓ post-quantum

Recursive STARK

input*:

(input, program, output, PROOF)

Recursive STARK

input*:

(input, program, output, PROOF)

program*:

verifier

Recursive STARK

input*:	program*:	output*:
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Recursive STARK

input*: (input, program, output, PROOF)

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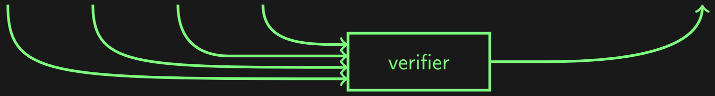
output*: 0/1



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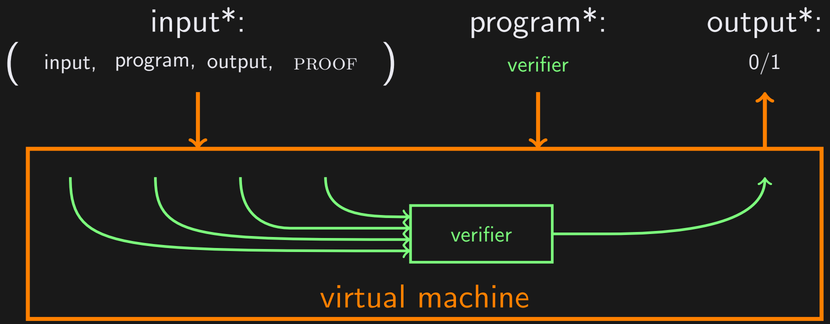

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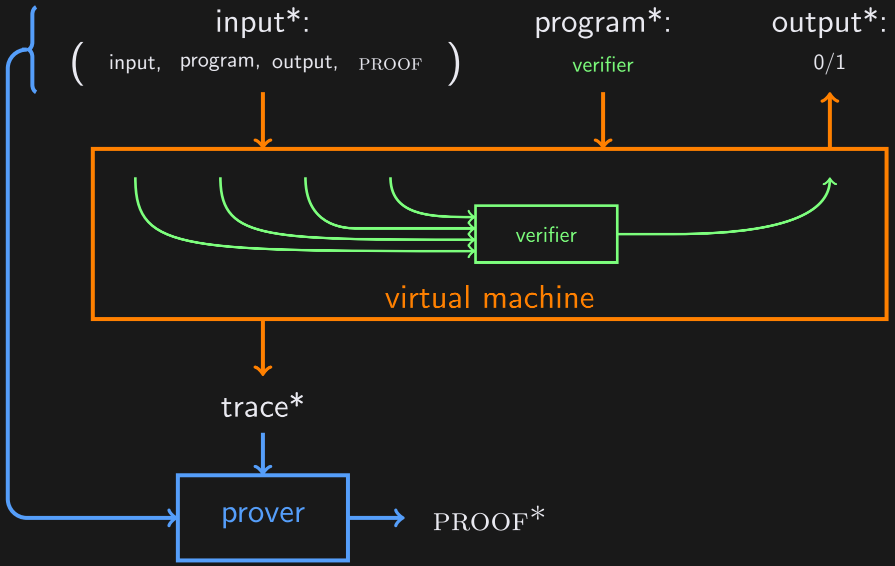

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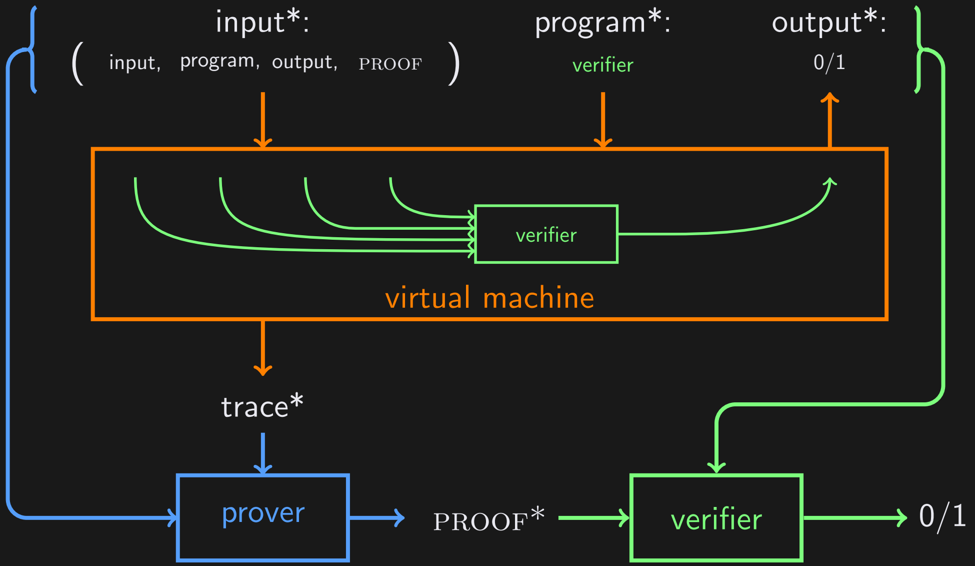
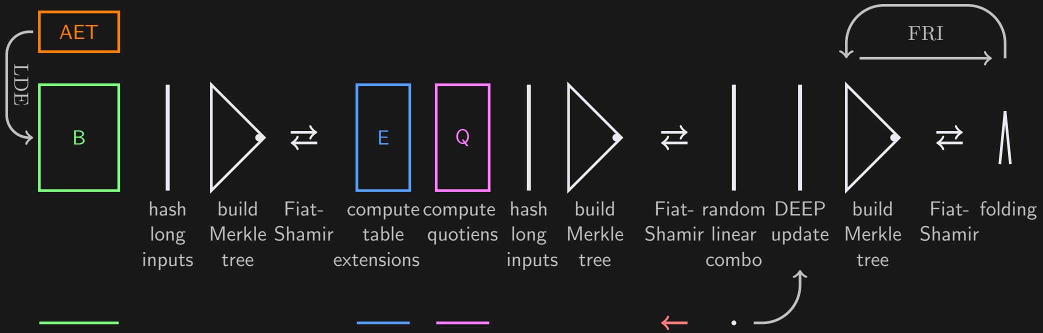


STARK Workflow



STARK Cost

Prover:

- LDE
- trace arithmetic
- hashing long inputs
- building Merkle tree
- Fiat-Shamir
- evaluate AIR
- hashing long inputs
- building Merkle tree
- Fiat-Shamir
- out-of-domain evaluation
- evaluate AIR
- random linear combination
- DEEP update
- FRI

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... but 80% of the time is spent hashing.

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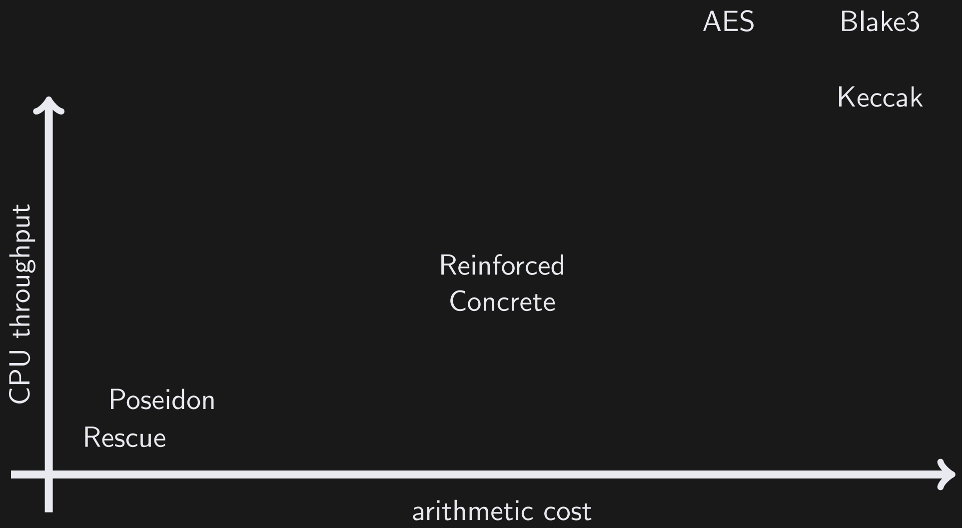
The VM must support hashing.

$\Rightarrow$ **we need an arithmetization-friendly hash function.**

Hash Function Orientation



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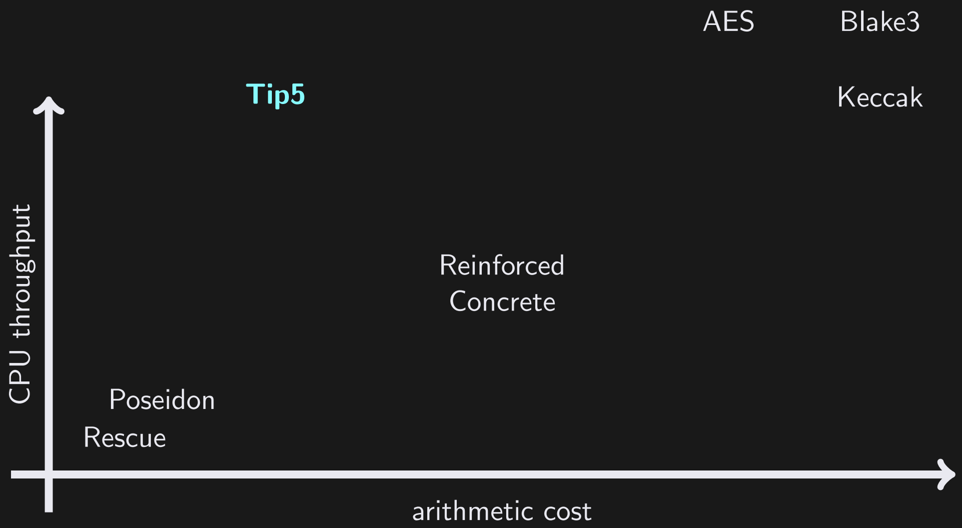

Security of Arithmetization-Oriented Hash Functions

1. Statistical Cryptanalysis ✓✓✓
2. Algebraic Cryptanalysis ?????
   - in particular, *Gröbner basis* algorithms

Interlude: Gröbner Basis Algorithms

$$\left.\begin{aligned} x^3 - xy + 2y^2 - x + 1 = 0 \\ y^3 + 2x^2 + y^2 - x - y - 1 = 0 \end{aligned}\right\}$$

Interlude: Gröbner Basis Algorithms

$$\left.\begin{array}{r} x^3 - xy + 2y^2 - x + 1 = 0 \\ y^3 + 2x^2 + y^2 - x - y - 1 = 0 \end{array}\right\} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 & 0 & 1 & -1 & -1 & -1 \end{pmatrix}$$

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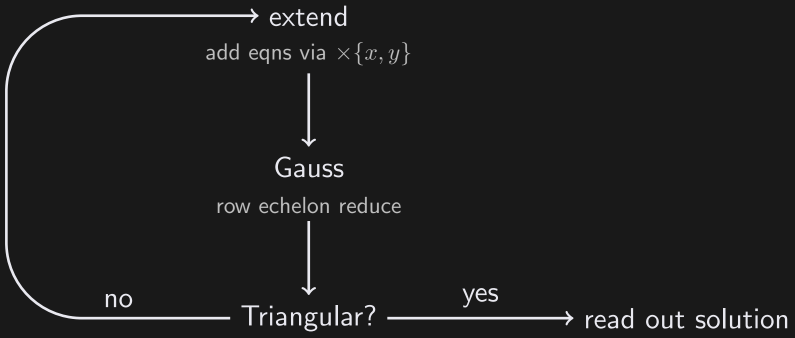

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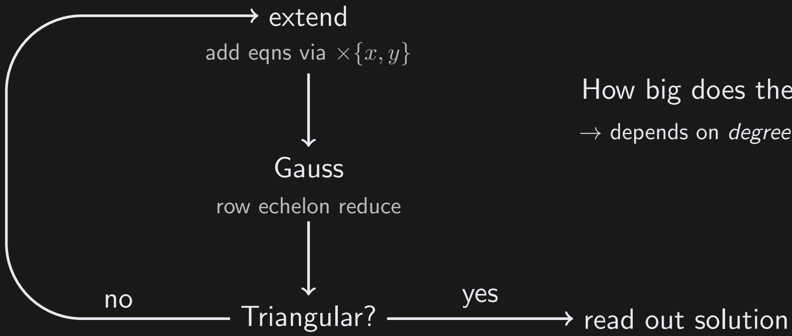

How big does the matrix get?

$\rightarrow$ depends on *degree of regularity*

Gröbner Basis Algorithms and Arithmetization-Oriented Hash Functions

1. Hash function is arithmetization-oriented
   $\Rightarrow$ it has a compact multivariate polynomial description
   $\Rightarrow$ Gröbner basis algorithms are relevant

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4. Reinforced Concrete
   - **lookup gates seem to defy GB attacks**

Design Criteria

Reinforced Concrete:

We Need:

Design Criteria

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- $p \in \{p_{\mathrm{BLS381}}, p_{\mathrm{BN254}}, p_{\mathrm{ST}}\}$

We Need:

- $p = 2^{64} - 2^{32} + 1$

Design Criteria

Reinforced Concrete:

- $p \in \{p_{\text{BLS381}}, p_{\text{BN254}}, p_{\text{ST}}\}$
- non-uniform rounds:
  $(C \circ B)^n \circ C \circ \text{Bars} \circ C \circ (B \circ C)^n$

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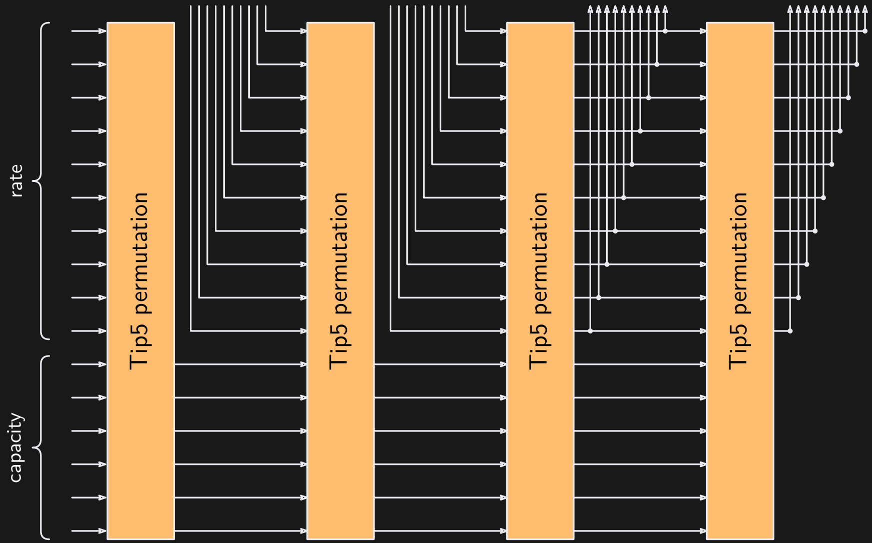
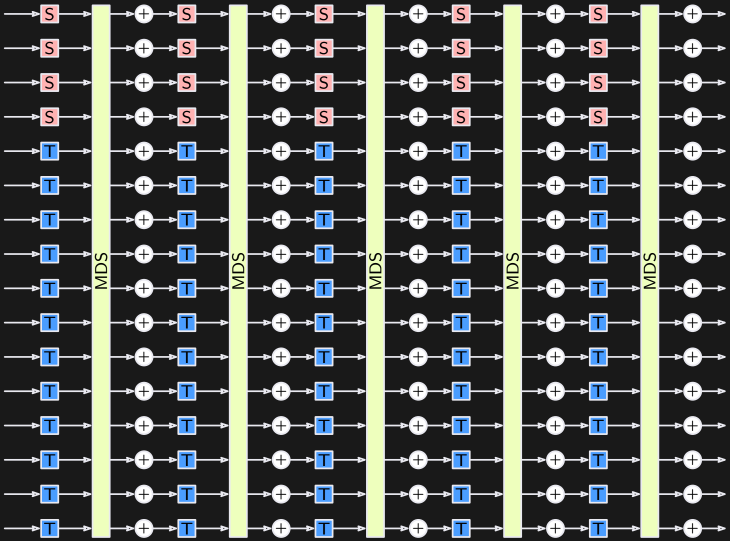
↑

Tip5 design constraints

Tip5 Sponge Construction



Tip5 – Permutation



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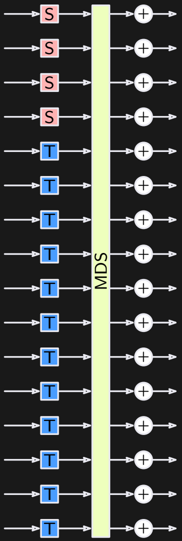

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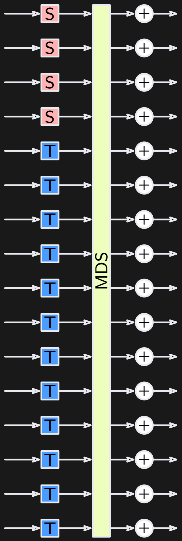

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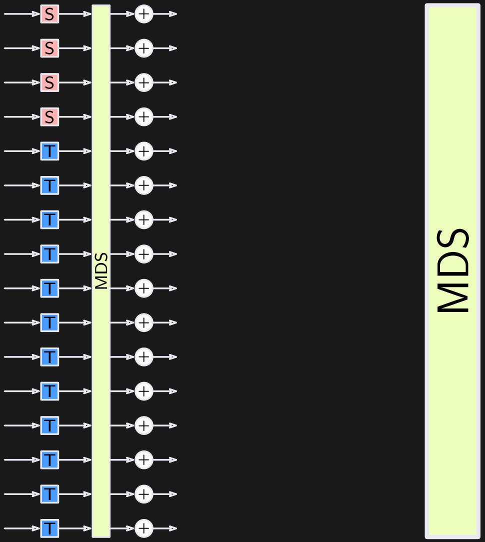

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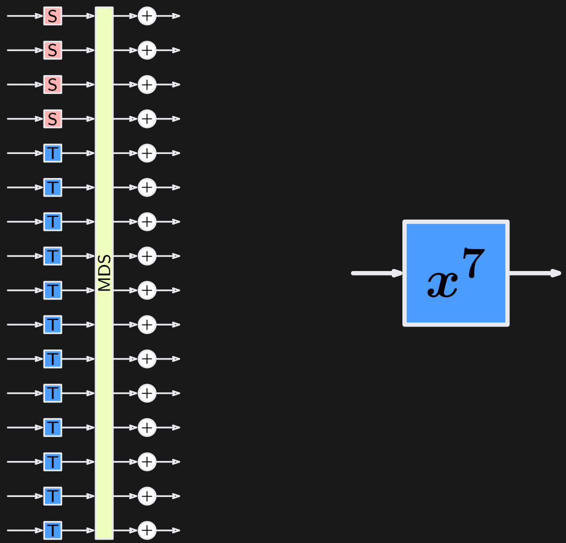

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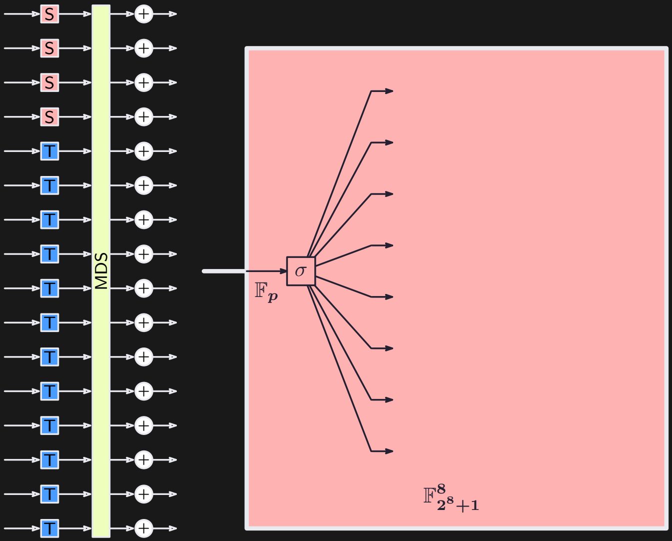

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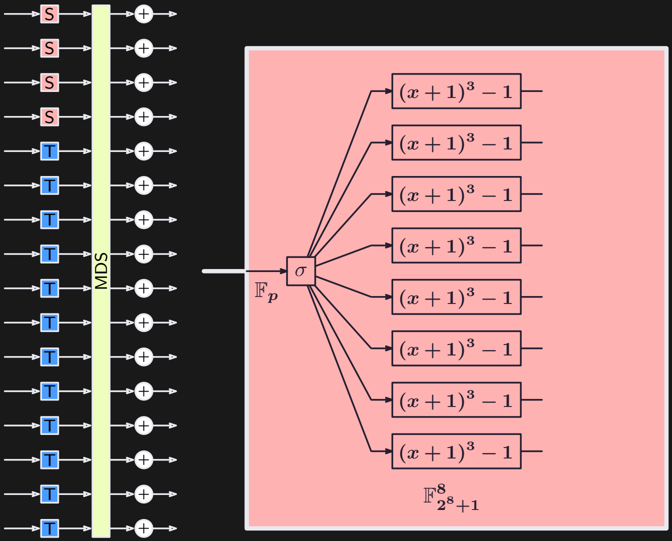

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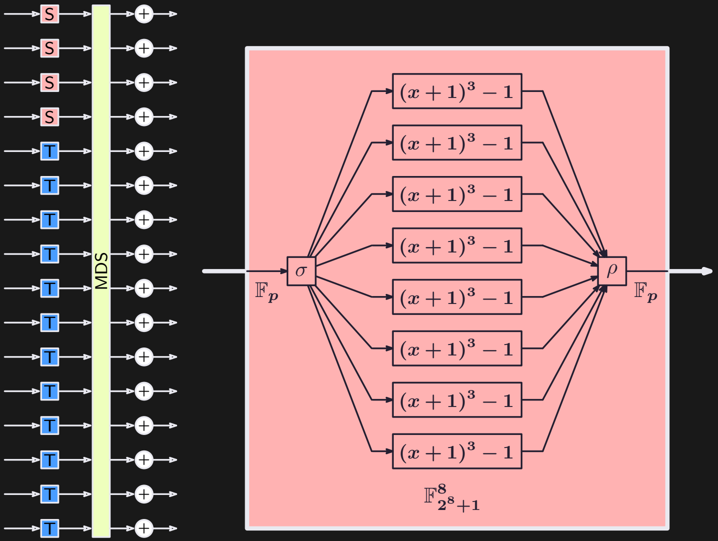

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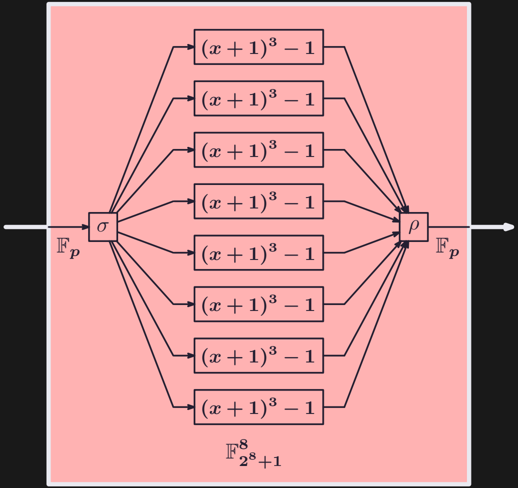

$L : \mathbb{F}_{257} \to \mathbb{F}_{257}\,,\;\; x \mapsto (x+1)^3 - 1$

– is a permutation on F_{257}

– has fixed points $\{0, 255, 256\}$

$\Rightarrow$ also a permutation on $\{0, \ldots, 255\}$

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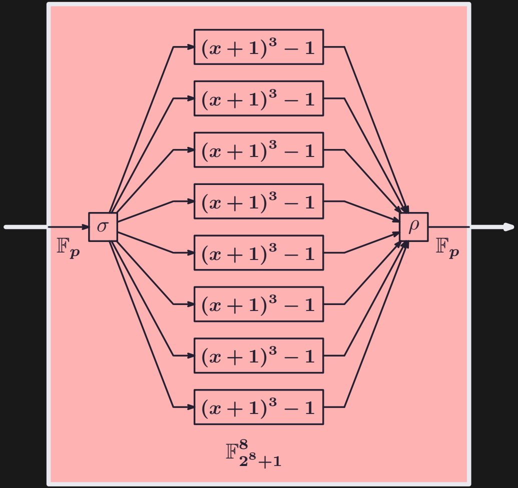

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Consider $S = \sigma \circ L^8 \circ \rho$ as a map on $\mathbb{N}_{<2^{64}}$

- $x = p - 1 \Leftrightarrow x =$ `0xffffffff00000000` maps to `0xffffffff00000000` $= p - 1$
- $x \geqslant p \Leftrightarrow x =$ `0xffffffff********` maps to `0xffffffff********` $\geqslant p$

$\Rightarrow x < p$ maps to $S(x) < p$ $\square$

Circulant MDS Matrix (1)

$$\begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix} \begin{pmatrix} i \\ j \\ k \end{pmatrix} \quad \leftrightarrow \quad (a + bX + cX^2) \times (i + jX + kX^2) \mod X^3 - 1$$

Circulant MDS Matrix (2)

compute matrix-vector product *over the integers*

$\rightarrow$ avoids modular reduction

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→ avoids modular reduction

state vector first column

$$f \quad \times \quad g \quad \mod X^{16} - 1$$

$$\hookrightarrow f \times g \mod X^8 + 1$$

$$\hookrightarrow f \times g \mod X^8 - 1$$

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$$f \quad \times \quad g \qquad \bmod X^{16} - 1$$

$\hookrightarrow$ $f \times g \mod X^8 + 1 \longrightarrow$ Karatsuba

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  - $f \times g \mod X^4 + 1$
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$$f \quad \times \quad g \qquad \bmod X^{16}-1$$

- $\longrightarrow f \times g \bmod X^8+1 \quad \longrightarrow \quad$ Karatsuba
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  - $\longrightarrow f \times g \bmod X^4+1 \quad \longrightarrow \quad$ Karatsuba
  - $\longrightarrow f \times g \bmod X^4-1$
    - $\longrightarrow \ldots$
    - $\longrightarrow \ldots$

LogUp

Client

input	output
1	a
2	b
3	c
1	a

Server

input	output
1	a
2	b
3	c
4	d

LogUp

Client

input	output	comboc
1	a	$1 + a\alpha$
2	b	$2 + b\alpha$
3	c	$3 + c\alpha$
1	a	$1 + a\alpha$

Server

input	output	combos
1	a	$1 + a\alpha$
2	b	$2 + b\alpha$
3	c	$3 + c\alpha$
4	d	$1 + a\alpha$

$$\texttt{combo} = \texttt{input} + \alpha \cdot \texttt{output}$$

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2	b	$2 + b\alpha$
3	c	$3 + c\alpha$
1	a	$1 + a\alpha$

Server

input	output	combos	mult.
1	a	$1 + a\alpha$	2
2	b	$2 + b\alpha$	1
3	c	$3 + c\alpha$	1
4	d	$1 + a\alpha$	0

$$\texttt{combo} = \texttt{input} + \alpha \cdot \texttt{output}$$

LogUp

Client

input	output	comboc	ldc
1	a	$1+a\alpha$	$-$
2	b	$2+b\alpha$	$-$
3	c	$3+c\alpha$	$-$
1	a	$1+a\alpha$	$-$

Server

input	output	combos	mult.
1	a	$1+a\alpha$	2
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3	c	$3+c\alpha$	1
4	d	$1+a\alpha$	0

$$\texttt{combo} = \texttt{input} + \alpha\cdot\ \texttt{output}$$

$$\texttt{ldc}_i = \texttt{ldc}_{i-1} + \frac{1}{\beta - \texttt{comboc}_i} \text{ and } \texttt{ldc}_0 = \frac{1}{\beta - \texttt{comboc}_0}$$

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1	a	$1 + a\alpha$	–
2	b	$2 + b\alpha$	–
3	c	$3 + c\alpha$	–
1	a	$1 + a\alpha$	–

Server

input	output	combos	mult.	sum
1	a	$1 + a\alpha$	2	–
2	b	$2 + b\alpha$	1	–
3	c	$3 + c\alpha$	1	–
4	d	$1 + a\alpha$	0	–

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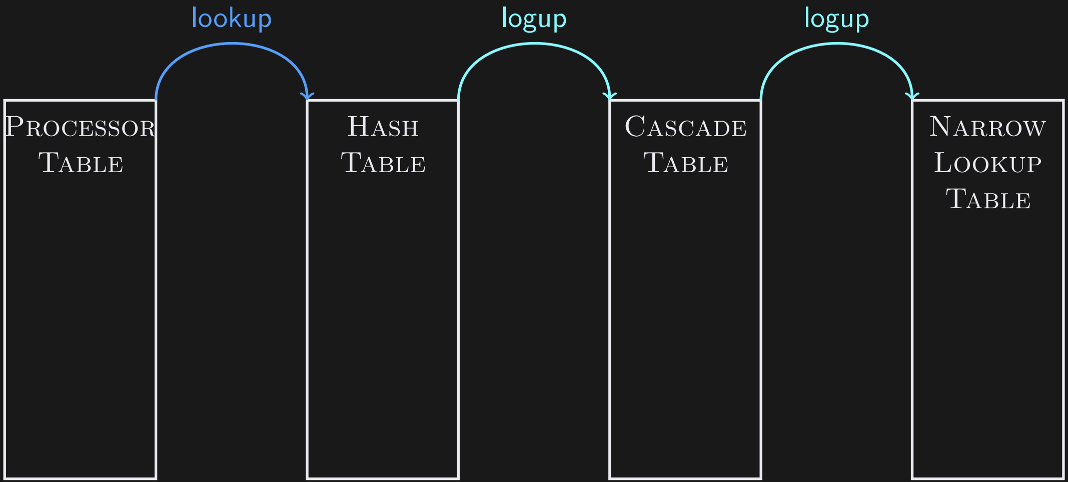
$$\sum_i \frac{1}{\beta - \texttt{comboc}_i} = \sum_i \frac{\texttt{mult}_i}{\beta - \texttt{combos}_i}$$

Cascade


lookup
logup
logup
Processor Table
Hash Table
Cascade Table
Narrow Lookup Table

Conclusion

Hash Function	Time [μs]
Rescue-Prime	18.186
Rescue-Prime Optimized	14.357
Poseidon	6.825
Tip5	0.850

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$2.68\times$ speedup in **Triton VM**.

The Tip5 Hash Function for Recursive STARKs

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Polygon

neptune

Triton VM